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Cryogenic flow rate measurement with a laser Doppler velocimetry standard

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Abstract
A very promising alternative to the state-of-the-art static volume measurements for liquefied natural gas (LNG) custody transfer processes is the dynamic principle of flow metering. As the Designated Institute (DI) of the LNE (‘Laboratoire National de métrologie et d’Essais’, being the French National Metrology Institute) for high-pressure gas flow metering, Cesame–Exadebit is involved in various research and development programs. Within the framework of the first (2010–2013) and second (2014–2017) EURAMET Joint Research Project (JRP), named ‘Metrological support for LNG custody transfer and transport fuel applications’, Cesame–Exadebit explored a novel cryogenic flow metering technology using laser Doppler velocimetry (LDV) as an alternative to ultrasonic and Coriolis flow metering.

Cesame–Exadebit is trying to develop this technique as a primary standard for cryogenic flow meters. Currently, cryogenic flow meters are calibrated at ambient temperatures with water. Results are then extrapolated to be in the Reynolds number range of real applications. The LDV standard offers a unique capability to perform online calibration of cryogenic flow meters in real conditions (temperature, pressure, piping and real flow disturbances). The primary reference has been tested on an industrial process in a LNG terminal during truck refuelling. The reference can calibrate Coriolis flow meters being used daily with all the real environmental constraints, and its utilisation is transparent for LNG terminal operators.

The standard is traceable to Standard International units and the combined extended uncertainties have been determined and estimated to be lower than 0.6% (an ongoing improvement to reducing the correlation function uncertainty, which has a major impact in the uncertainty estimation).

Keywords: aerodynamics, LNG, primary standard, LDV, cryogenics, calibration, on-site measurements

(Some figures may appear in colour only in the online journal)
Temperature at measuring conditions

$D$, $V$, $E$

and for NOx fuel to meet the new limits for sulphur content in marine fuels is particularly suited for long-distance road freight transport for which alternatives to diesel are limited. Furthermore, engines running on LNG produce much less noise than diesel-operated engines. The application of LNG as a fuel is one of the pillars of its Clean Fuel Strategy. LNG is also an attractive choice for deliveries in urban areas. LNG is very much welcomed by the European Commission as the use of LNG as a transport fuel is gaining wider attention and within Europe LNG volume quantity is being measured on-ship using tank level measurement systems in combination with tank calibration tables. In comparison with other commodities like natural gas or gasoline, the total uncertainty of measured energy is high for LNG and has been estimated to be up to 1% (see Kerkhof [2]). When the ship suffers from wave motion (off shore), the measurement becomes even more difficult. The tank calibration creates an additional challenge.

The aim of the related JRP is to further develop the metrological framework for LNG, for both small and large-scale applications. This will lead to a significant reduction of uncertainties in the determination of transferred energy in LNG custody transfer processes. Cesame–Exadébit has been involved in the development and validation of novel and traceable calibration standards of LNG mass and volume flow for vehicle fuel dispensing and ship bunkering. To measure LNG volume, we studied the capability of a flow measurement system using laser Doppler velocimetry (LDV) (see Strzelecki et al [3]). The LDV system may provide an alternative traceability route for LNG volume flow meters. It can also be used as a non-intrusive instrument to perform flow profile measurements in a cryogenic medium.

LDV as a flow measurement technology has already been demonstrated under high pressure with natural gas (5.5 MPa) with an uncertainty of 0.22% (see Mickan et al [4]) but its extension to cryogenic temperatures is especially challenging because

1. The measurement of a velocity component in LNG by means of LDV requires the introduction into the fluid through one optical path of two laser beams which intersect to form the measurement volume. There are two features to be taken into account: the size of the volume measurement has to be calculated in order to capture the main aerodynamic features of the flow and most importantly, the fluid temperature is around $-160 \, ^\circ C$ whereas the flow measurement system is at ambient temperature. Icing on the optical path must be prevented in order to perform an accurate velocity measurement.

2. The LDV measurement requires the presence of micron-size tracers in the fluid. In the absence of natural micron-size tracers, it is essential to provide a clean seeding system that does not contaminate the fluid.

3. The objective is to measure instantaneous LNG flow rate with an uncertainty around 0.2% with traceability of the measurement.

This present paper is organized as follows. Section 2 presents the technical solution to perform a cryogenic measurement with LDV system (optical windows, vacuum insulation, seeding, etc). Section 3 is focused on the LDV measurement technique explanation. Section 4 is devoted to the assessment...
of the shear layer dependence on the mass flow with a simplified measurement package by means of experiments conducted with air. Section 5 presents experiments that have been realized in cryogenic conditions in the National Institute of Standard and Technology (NIST) in Boulder, Colorado using Liquid Nitrogen. Section 6 gives some information regarding on-site experiments in an LNG terminal during truck filling. Finally, Section 7 gives some conclusions and perspectives for future work. The uncertainty budget assessment in cryogenic conditions has been detailed in the appendix.

2. Experimental setup and LDV package description

2.1. Description of Cesame reference facility for flow rate measurements

The pressurized calibration facility for medium and high flow rates at Cesame–Exadebit can generate flow rates from $8 \text{ m}^3 \text{ h}^{-1}$ to $80\,000 \text{ m}^3 \text{ h}^{-1}$ (normal conditions). A set of twelve Venturi nozzles (nominal flow rate: 1.5 to $1000 \text{ m}^3 \text{ h}^{-1} \text{ bar}^{-1}$) operating in sonic conditions is used for the determination of the standard mass flow rate. The test pressure range is from 1 bar up to 45 bar (absolute). Compressed dry air stored in a 110 m$^3$ vessel under 200 bar (absolute) is used as the test fluid. The air coming from the storage vessel goes through the valves and the heating control system. A suitable temperature and pressure upstream of the nozzles is automatically adjusted.

This configuration allows a comparison between the reference and tested device mass flows. The pressure and the temperature can be measured at the level of the meter under test in order to determine the volume flow rate going through it. The real gas effects are taken into account by applying compressibility factor corrections to the thermodynamic conditions where the measurement is taken. These nozzles are traceable to National Standards by mean of a $(P, V, T, \text{time})$ method (pressure, volume, temperature and time). More information regarding this calibration technique can be founded in Wright et al [5].

2.2. Description of the DN80 cryogenic LDV measurement package

The LDV measurement package is composed of three main sections: (1) the cryogenic seeding part, (2) the conditioning part with the measuring cross-section and (3) the divergent (see figures 2(a) and (b)).

The seeding part is equipped with an access for the seeding probes in cryogenic conditions, and with two windows for particle visualization (see figure 2(b)). The conditioning part is provided with windows which allow passage of laser beams for measuring the velocity profile at the exit of the convergent. The downstream part of the cryogenic LDV measurement package contains the divergent. These three parts are located inside a vacuum chamber to ensure thermal insulation.

The main characteristics of the cryogenic LDV system are described in more details in Strzelecki et al [6]. These are briefly restated below:

- Internal diameter $D = 80 \text{ mm}$
- Throat diameter $d = 40 \text{ mm}$
- Beta ratio of the convergent $d/D = 0.5$

![Figure 1](https://via.placeholder.com/150)

Figure 1. LNG supply–demand gap opens post-2020, from Cheniere interpretation of Wood Mackenzie data (Q4 2016).

![Figure 2](https://via.placeholder.com/150)

Figure 2. (a) Description of the simplified cryogenic LDV measurement package and (b) 2D model horizontal cross-sectional view.
The seeding unit is equipped with an access for external seeding forcing (probes under cryogenic conditions) and with two portholes for particle visualization. Indeed, LDV requires micron-size particle injection to achieve velocity measurements (if particles are not naturally in the tested fluid). Generally, there is enough material (micron-sized particles) to reach a statistical convergence of the mean and fluctuating velocities. Nevertheless, an external seeding might be needed to insure a higher and quicker statistical convergence. The buoyancy of the seeding must be relevant to the main flow. The surface stresses acting on the particle are very important to the dynamics. The choice of the seeding system is important regarding the measurement accuracy. Two alternative systems can be used. First, injection of micron-size bubbles upstream of the LDV measurement volume by local boiling of LNG using a small electrical current and second, injection of a gas or liquid with a low flow rate to generate solid micron-size particles at cryogenic temperature (argon could be a candidate—solidification at 110 K with 5 bar).

The LDV measurement unit consists of an optimized convergent for conditioning the cryogenic flow before measuring the local velocity at the throat section by means of the LDV. Indeed, the shape of this convergent standardizes the mainstream and provides a thin axial velocity gradient at the nozzle exit.

To process the LDV measurement, it is necessary to introduce into the model dual laser beams that intersect at the measurement volume. It must be moved across the throat section of the convergent to provide the velocity profile. These laser beams are introduced through two specific portholes. The first one is an interface between the cryogenic liquid (−160 °C, pressure < 10 bar) and the insulation vacuum chamber (pressure at 10⁻⁵ Torr).

Figure 3 shows the diagram of the optical configuration with the laser beams.

The second porthole is an interface between the insulation vacuum chamber and the ambient atmospheric conditions. This assembly needs a very accurate spatial positioning to maintain the parallelism which provides beam intersection in the direction perpendicular to the flow. In addition, these portholes must withstand a large variation of temperature and pressure.

3. Axial velocity measurement by LDV: measurement principle

The method of determining the volume flow rate from a local velocity (measured downstream of a throat) is presented in this section. The basic idea is to design a flow conditioner (convergent) that provides a symmetrical and flat velocity profile in order to assure a very repeatable and fast profile measurement. Once the boundary layers are analytically determined or directly measured, the flow rate measurement can be reduced to a single point measurement (centreline axial velocity measurement).

\[ Q_v = \pi \frac{d^2}{4} \nu \]  

To be a primary standard, a theoretical framework needs to be developed to accurately determine the shear layer influence on the mass flow rate assessment (with a single point measurement in the centreline axis). As an example, if the velocity profile is a piston profile, the shear layer influence is negligible whereas in case of channel flow or free jet, the Reynolds stresses modify the velocity magnitude over the diameter significantly.

Cesame–Exadebit is currently working towards proposing this equipment as a primary standard for cryogenic flow meters. It is a challenging objective and Cesame needs to continue its journey to define the best theoretical approach to achieve this shear layer evaluation. In the meantime, the shear layer influence can also be determined experimentally by using a comparison with another standard. The LDV package then becomes a secondary standard. In the following chapters of this paper, this approach will be presented.

The velocity profiles are measured by means of a LDV from DANTEC in the backscattering mode with the following specifications. The laser wavelength is 532 nm (green beams) with a 160 mm focal lens. The fringe spacing is 2.217 ± 0.026 μm. The volume measurement is 0.41 mm × 0.05 mm. Further information regarding the LDV principle can be founded in Eder et al [7].

The laser is traceable to SI units by the Doppler frequency and the fringe spacing. The velocity is calculated using the equation below:

\[ \nu = \frac{I(f_D - f_S)}{\cos(\gamma)} \]  

where \( I \) is the fringe spacing, \( f_D \) is the Doppler frequency measured by the system as the particle flows through the measuring volume, \( f_S \) is a frequency shift deliberately added to one of the beams in order to retrieve directional information (see e.g. [7] for more details) and, \( \gamma \) is the offset from 90° at the measurement spot. The fringe spacing (\( I \)) of the LDV system has been calibrated by a notified body (CETIAT in France).

The volume flow rate is obtained by measuring the mean velocity (\( \bar{\nu} \)) at the throat (if the velocity gradient has a small impact on the average flowrate). The governing equation can be written as equation (3):

\[ Q_v = \pi \frac{d^2}{4} \bar{\nu}. \]
Figure 4. Evolution of the growing of the shear layer and momentum thickness. Direct link between single velocity point measurement to volume mass flow assessment.

Table 1. Measurement conditions during experimental phase.

<table>
<thead>
<tr>
<th>Upstream pressure $P$ (bar)</th>
<th>Nominal velocity $v$ (m s$^{-1}$)</th>
<th>Reynolds number $Re_d$</th>
<th>Mass flowrate $Q_m$ (kg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11.3</td>
<td>5.95 $\times$ 10$^4$</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>21.4</td>
<td>1.13 $\times$ 10$^5$</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>57.9</td>
<td>3.12 $\times$ 10$^5$</td>
<td>0.176</td>
</tr>
<tr>
<td>5</td>
<td>10.8</td>
<td>1.44 $\times$ 10$^5$</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>21.1</td>
<td>2.81 $\times$ 10$^5$</td>
<td>0.159</td>
</tr>
<tr>
<td></td>
<td>58.8</td>
<td>7.61 $\times$ 10$^5$</td>
<td>0.432</td>
</tr>
<tr>
<td>10</td>
<td>10.5</td>
<td>2.78 $\times$ 10$^5$</td>
<td>0.158</td>
</tr>
<tr>
<td></td>
<td>20.5</td>
<td>5.40 $\times$ 10$^5$</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>58.5</td>
<td>1.56 $\times$ 10$^6$</td>
<td>0.862</td>
</tr>
</tbody>
</table>

Furthermore, the ratio between the velocity on the axis $v_{\text{axis}}$ (measured by the LDV system) and the output of the mean velocity $\bar{v}$ is a constant value for each Reynolds number based on the throat diameter $d$. The function combining all the values is defined as $A^*$ and the expression can be written as

$$\frac{v_{\text{axis}}}{\bar{v}} = A^*(Re_d).$$  \hspace{1cm} (4)

Where for a circular cross section duct of diameter $d$,

$$Re_d = \frac{vd}{\mu} = \frac{4Q_m}{\pi \mu d^4}.$$  \hspace{1cm} (5)

These relations allow the calculation of the volume flow rate from the axial velocity measured at one point downstream of the throat on the centreline axis. It can be written as follows:

$$Q_s = \frac{\pi d^2}{4} \bar{v} = \frac{\pi d^2}{4} \frac{v_{\text{axis}}}{A^*(Re_d)}.$$  \hspace{1cm} (6)

To implement this method, it is mandatory to establish a transposition function between the volume flow rate and the local velocity measurement (as a function of Reynolds number). Ideally, if the axial velocity profile was piston-type or top-hat shape (at the nozzle lip), the transposition from local centreline velocity to volume flow rate will be straightforward. Indeed, the momentum thickness of the shear layer will have no influence ($\delta_\omega \rightarrow 0$). The mass flow rate will be the integration of the velocity on the centerline over the diameter. The figure 4 below shows the growing of the shear layer.

The stream-wise velocity profile in this region resembles a hyperbolic tangent function. The figure above demonstrates this hypothesis. The stream-wise profile can be assessed by

$$U(y) = \frac{U_1 + U_2}{2} + \frac{\Delta U}{2} + \tan \left( \frac{2y}{\delta_\omega(0)} \right).$$  \hspace{1cm} (7)

From the boundary layer evolution theory, the growing of the vortices downstream the nozzle lead to an increase of the momentum thickness of the shear layer ($\delta_\omega$) and the transposition for single velocity point measurement to volume mass flow needs to be adjusted by a calibration factor (i.e. $A^*$ function (depends to $Re_d$)) to takes this growing into account. The key parameter in this situation is the momentum thickness of the shear layer.

4. Experimental phase: air-based experiments

4.1. Test matrix

During these tests, the Reynolds number increased from $6 \times 10^4$ to $1.5 \times 10^6$. The table 1 shows the nominal conditions:

4.2. Single point measurement in the irrotational zone (jet centreline axis)

The single point measurement has the main advantage of being quick and continuous. It is more convenient for industrial partners since this concept will directly be mounted on site with their real experimental conditions. The flow meter will calibrate other kind of measurement system which are currently calibrated with water (Coriolis or ultrasonic meters over a small range of mass flow rates). This concept does not have technical limitations with respect to the on-site range of mass flow (adjustments for pipe size might have to be made).

The goal of this experimental phase is to have a one-point measurement to determine the correlation function $A$ for a large range of Reynolds numbers (with air). The single point measurement is taken in the potential core which is quasi-irrotational.

During these experiments, a turbine was mounted downstream of the flow meter in order to calculate the complete mass flow rate. Indeed, the reference mass flow is given by the sonic nozzle but it does not take into account the seeding
1.5 \times 10^6. The correlation function is defined in this range from calibration.

Reynolds number (air measurement) with extended uncertainties of Reynolds numbers from 1 \times 10^5 to 1.5 \times 10^6. The correlation function is defined in this range of ReD due to facility limitation. The upstream pressure was raised from 5 to 10 bar to reach the highest Reynolds number. The results of the experimental phase are summarized in the table below. Physical parameters such as pressure, mass flow of the standard facility and our flow meter are also restated.

As stated earlier, a direct comparison of the mass flow of sonic nozzles and the LDV package cannot be done since the seeding is not taken into account with the standard facility. However, we can determine the mean axial velocity from the turbine mass flow. It is then possible to compare the direct single point velocity measurement with the latter quantity. Figure 5 shows the evolution of the correlation function (A') regarding the increasing Reynolds number.

The figure shows that the slope of the fit is monotonous and looks to be logarithmic. The Y error bars are the extended uncertainty obtained during turbine calibration (made on the same test rig with sonic nozzles as a transfer standard: combined extended uncertainties being lower than 0.25%).

Figure 5 also shows that the correlation function influence decreases when the Reynolds number increases. Indeed, the momentum thickness (δω) is significantly reduced. An analytical function has been determined by iteration and its expression is stated below:

\[ y = a + b \cdot \log(x). \]  

(8)

The single point measurement in air provides a database which allows the determination of a correlation function to accurately determine the mass flow volume rate over a range of Reynolds numbers from 1 \times 10^5 to 1.5 \times 10^6. This measurement system offers unique advantages such as

1. On-site calibration of industrial flow meters which are operating every day,

2. Real test conditions (temperature, pressure and fluid properties),

3. No limitation in terms of mass flow rate (piping needs to be adjusted),

4. Continuous velocity measurements with the same accuracy during transition (to reach the target velocity, for example).

Cesame–Exadebit needs to obtain confirmation that the trends observed with air measurements are still relevant to cryogenic conditions. Low temperature concerns are important and Cesame–Exadebit has to verify numerous assumptions made for designing the LDV package. The next section is devoted to this matter.

5. Experimental phase: nitrogen-based experiments

5.1. Experimental setup and test plan

The LDV package test was run on the NIST cryogenic flow measurement facility. This facility has a combined uncertainty in the measurement of 0.18% for the totalized volume flow (k = 2) (see Scott et al [8]). This uncertainty statement applies to measurements made within a flow range of 75 to 750 l min\(^{-1}\). NIST have incorporated a newer equation of state for nitrogen (see Span et al [9]), and the uncertainty in density for the new equation is 0.02% (k = 2).

A rangeability test was run to determine flow meter performance over a range of flow rates and fixed temperature and pressure. The temperature was about 80 K, the pressure range was about 5 bar. There were five separate flow rates selected which were repeated at least three times. During this experimental phase, Cesame–Exadebit only tried to calculate mass flow rate by using single point measurement in the jet centre-line axis. The test plan is restated in table 3 below.

5.2. Objectives of the nitrogen phase

This phase gave Cesame–Exadebit the opportunity to confirm that the LDV package can operate in cryogenic conditions, to allow validation of the technical choices and to test numerous hypotheses regarding the cryogenic conditions such as mechanical behaviour in cryogenic conditions, the vacuum level required to perform velocity measurements without icing on portholes, the optical convergence of the beams with liquid nitrogen or the instrumentation permeability with cryogenic fluid.

<table>
<thead>
<tr>
<th>Run</th>
<th>(Q_{m0}) (kg s(^{-1}))</th>
<th>Throat velocity (m s(^{-1}))</th>
<th>Mach number</th>
<th>ReD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.00</td>
<td>2.0</td>
<td>0.005</td>
<td>4.36 \times 10^5</td>
</tr>
<tr>
<td>2</td>
<td>2.46</td>
<td>2.5</td>
<td>0.007</td>
<td>5.37 \times 10^5</td>
</tr>
<tr>
<td>3</td>
<td>3.55</td>
<td>3.5</td>
<td>0.010</td>
<td>7.73 \times 10^5</td>
</tr>
<tr>
<td>4</td>
<td>4.91</td>
<td>4.9</td>
<td>0.014</td>
<td>1.07 \times 10^6</td>
</tr>
<tr>
<td>5</td>
<td>7.11</td>
<td>7.1</td>
<td>0.020</td>
<td>1.55 \times 10^6</td>
</tr>
<tr>
<td>6</td>
<td>9.17</td>
<td>9.2</td>
<td>0.027</td>
<td>2.00 \times 10^6</td>
</tr>
</tbody>
</table>
5.3. Single point measurement: results and discussion

The laser beam convergence was determined with accuracy by taking into account the length modification of the convergence beams due to nitrogen refraction indices. Convergence positioning (during experiments) was controlled using a remote camera mounted on one of the optical access points of the LDV package (on the top) since no one can be close to the laser during experiments (safety procedures for a class 4 laser). The laser system was not moved in order to ensure the convergence of the beams to the centreline axis close to the throat \( \frac{x}{D} = 0.25 \) approximately. The experimental setup is visible in figure 6:

The main goal of these tests was to determine the correlation function \( \Phi^*(Re) \) in cryogenic conditions (the same procedure carried out during air based experiments). During these experiments, the upstream pressure was fixed around 5 bar and the velocity was increased from 2 to 9 m s\(^{-1}\) resulting in a Reynolds number based on the throat diameter from \( 4.3 \times 10^5 \) to \( 2.0 \times 10^6 \). Each mass flow rate has been repeated at least three times to access standard deviation in the laser results. A trigger signal was sent from the NIST acquisition system to our laser/sensors acquisition system to provide a relevant set of data for performing the measurements and calculations.

Cesame–Exadebit applied the correlation function determined during the air-based experimental phase in Poitiers. The Reynolds number is the similitude criterion. Even if there is a fluid property modification (from air to cryogenic liquid), the correlation is not expected to be highly modified. Nevertheless, this NIST phase provided a new correlation function for cryogenic conditions that Cesame will test soon.

Cesame must take into account the low temperature constraints on the flow meter body. Indeed, the reduction of the throat diameter due to cryogenic temperature affects largely the assessment of the axial velocity. The paper of Thermeau \[11\] presents the length modification \( \frac{\Delta l}{l} \) as a function of temperature in Kelvin. In our case, the \( \Delta T \) is 213 K resulting in a length modification around 0.27% on the diameter.

Figure 7 presents the results of the bias in velocity between the standard facility and the LDV package. As a general comment, the trend observed during the air experiments is continued under cryogenic conditions since at low Reynolds numbers, the correlation function plays a higher role. The increasing Reynolds number leads to a reduction of the momentum thickness in the shear region. Figure 7 above shows that the comparison of the velocity between the standard facility and the LDV package is comprised between: \(-0.3\%\) and \(0.3\%\) with \(0.18\%\) of extended uncertainty. As a first attempt under cryogenic conditions, these results are really promising. Cesame–Exadebit wants to investigate further by having another experimental phase during which the correlation function defined under cryogenic conditions will be used to reduce the bias observed in figure 7.
5.4. Feedback on previous objectives

During these tests, many topics required investigation. Cesame–Exadebit’s knowledge of cryogenic conditions improved significantly during this experimental phase. The explanation for each topic is given below:

1. Mechanical behaviour in cryogenic conditions: the LDV package operated correctly throughout the experimental phase. No internal leakage was detected and the thermal constraints of the body part were correctly handled by all the seals/O rings used in the system.

2. Vacuum level required to perform velocity measurement without icing on portholes: the vacuum system operated effectively during the runs with a vacuum level around $1^{-4}$ mbar. This was enough to provide a sufficient vacuum level to avoid icing on the portholes.

3. Optical convergence of the beams with liquid nitrogen: the optical path had been calculated theoretically and it appears that it is relevant to what we saw during testing. The convergence of the beams allows us to perform LDV measurements with a high accuracy level.

4. Instrumentation permeability with cryogenic fluid: no leakage was detected during tests.

This method allows us to provide single point measurement velocity on the centreline axis of the jet over a large range of Reynolds numbers ($5 \times 10^4$ to $2 \times 10^6$). It can be used to calibrate on-site flow meters (Coriolis, ultrasonic flow meters, etc) which operate under cryogenic conditions. This technique has several advantages since it is much quicker and optical adjustments are much easier to make. The flow meters will be calibrated under real experimental conditions (temperature, pressure, fluid properties and large range of mass flow rates and Reynolds numbers).

5.5. Uncertainty budget evaluation

In this article and for illustration purposes, the uncertainty budget will be presented for a randomly-chosen measurement with liquid nitrogen at NIST under laboratory conditions. In this case, the uncertainty on mass flow rate has been estimated at 0.6% ($k = 2$) and the table 4 shows the contribution of each component. The full explanation of the budget can be found in the appendix.

![Figure 8. Diagrams of the LDV standard assembly (laser and vacuum enclosures, cryogenic mass flow rate standard and mobile frame).](image)

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$x_i$</th>
<th>Unit</th>
<th>$u(x_i)$</th>
<th>$c_i$</th>
<th>$u(y)$</th>
<th>Contrib (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>0.039 856</td>
<td>m</td>
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<td>0.000002</td>
<td>3</td>
</tr>
<tr>
<td>$A^*$</td>
<td>1.015</td>
<td>—</td>
<td>0.003 676 645</td>
<td>$-0.002 469 97$</td>
<td>$-0.000 009$</td>
<td>78</td>
</tr>
<tr>
<td>$v_{axis}$</td>
<td>2.04</td>
<td>m s$^{-1}$</td>
<td>0.003 654 776</td>
<td>0.001 229 05</td>
<td>0.000 004</td>
<td>19</td>
</tr>
<tr>
<td>$Y = Q_c$</td>
<td>0.002 507</td>
<td>m$^3$ s$^{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$U(y)/y = 0.31\%$

| $U(y)$ | $7.7 \times 10^{-6}$ |
| $k$ | 2.06 |
| $U(95\%)$ | $1.6 \times 10^{-5}$ |
| $U(y)/y$ | 0.63\% |

6. Cryogenic measurement in Montoir de Bretagne LNG terminal during a truck filling

Real-life, on-site calibration during truck filling requires obtaining accreditation from a notified body for explosive environments. The first task was to design a safe package that can be used in an LNG terminal (Ex IIGT6). Figure 8 below presents the diagram of our concept where both Atex enclosures (laser and vacuum), the flow meter and the frame can be seen. The LDV standard was set up within an industrial facility to measure the axial velocity between the LNG tank and the truck. The aim of these experiments was to compare both the weighing technique and a Coriolis flow meter. Figure 9 shows the experimental setup.

During this test, the installation and the exploitation of the LDV package was transparent for the LNG terminal (from the operator’s point of view). This equipment created no time delay for the truck refuelling. Cesame–Exadebit gained much...
useful experience on how to perform measurements in an industrial facility.

The natural micro-particles (micron-size particles) were detected in the LNG and the velocity measurements were consistent with the velocity calculated by the terminal output (truck scale, density, etc). There was no clear indication of the concentration, shape or numbers of these natural particles. Cesame–Exadebit is currently working to add an external seeding to maximize the capability of the LDV standard to be autonomic under any cryogenic setup.

7. Conclusions and perspective

Industry’s need to obtain traceability for cryogenic flowmeters has been outlined as an introduction to the work described in this paper. As a matter of fact, the economic impact of having accurate measurement of LNG is enormous since the global market is constantly increasing. The uncertainty on the volume of LNG is an important part of the overall uncertainty on the energy transfer from a carrier to a LNG terminal for example. Furthermore, current uncertainty assessments are rather optimistic and there is a lack of credibility from having non-representative conditions during flowmeter calibrations. Indeed, flowmeters are often currently calibrated in water at ambient conditions and calibration curves are then extrapolated to relevant Reynolds numbers for cryogenic application.

The work which has been done by Cesame–Exadebit within the Joint Research Program (JRP) on LNG serves to define a new route to the traceability chain. Within this framework, the LDV technique appears to be appropriate, since it is non-intrusive and the velocity can be traced back to SI unit by the fringe spacing and the Doppler frequency. The challenge to design suitable equipment for cryogenic conditions has been overcome (e.g. a vacuum required to avoid icing on the laser path, handling in an explosive environment). A set of experiments (in liquid nitrogen) has shown both promising results and the capability of the LDV standard to realise accurate velocity measurements. An exhaustive uncertainty budget in cryogenic conditions has been developed and it shows that the larger contribution can be attributed to the transposition function ($A^*$).

Finally, the LDV standard has been used during daily rotation of an LNG truck in a LNG terminal in Montoir de Bretagne. It is noteworthy that the carrying-out of the LDV experiments in a real-world environment during truck refuelling did not add difficulties or delays to the truck operations from an operator point of view at the LNG terminal in Montoir de Bretagne. The LDV standard offers a unique capability to perform online calibration of cryogenic flow meters under real conditions (temperature, pressure, piping and real flow disturbances) for end-users. The standard is traceable to SI units and the extended uncertainties have been determined and estimated to be lower than 0.6% (an ongoing improvement to reduce the transposition function uncertainty which has a major impact on the uncertainty estimation). Indeed, Cesame–Exadebit is currently working on an analytical model based on the boundary-layer theory (see Schlichting et al [10] for further explanations).

Acknowledgments

The research leading to the results discussed in this paper has received funding from the European Metrology Research Program (EMRP). The EMRP is jointly funded by the EMRP participating countries within Euramet and the European Union.

Cesame–Exadebit wants to thank Justervesenet—The Norwegian Metrology Service (and especially Åge Andreas Falnes Olsen and Gaël Chupin), for the inputs in building the cryogenic uncertainty budget.

Appendix

A.1. Cryogenic uncertainty budget assessment

The value of the measurand (the measurement) is obtained from a measurement of the time-averaged velocity on the
centreline axis of the LDV instrument. The axial velocity is converted to a cross-sectional average by applying a scaling factor obtained from a calibration curve (model function). The cross-sectional averaged velocity is then multiplied by the cross-sectional area to obtain the volumetric flow rate (see equation (3)).

Normally, the cross-sectional average velocity \( \overline{v} \) would be obtained by integration of the measured velocity profiles over the flow cross section. However, detailed information about the flow profile is difficult to obtain (in particular close to the wall) and it is time-consuming and challenging to measure it in real time. The LDV technique is therefore based on measuring the flow velocity at the flow centreline (pipe axis) only. An empirical scaling factor (derived from a model function) is applied to obtain the average velocity from the axial velocity.

In this work, the model function, that is the relationship between the axial velocity and the cross section average velocity, is approximated by a linear function of the natural logarithm of the Reynolds number (designated later as \( A' \)):

\[
\frac{v_{\text{axis}}}{\overline{v}} = a + b \ln (Re_a) + \epsilon = A^* \tag{A.1}
\]

where \( a \) is the intercept of the model function, \( b \) is the slope of the model function and, \( \epsilon \) is an error term introduced from assuming this particular relationship between \( v_{\text{axis}} / \overline{v} \) and \( \ln(Re_a) \).

Equations (3), (4) and (A.1) are combined to obtain the measurement function for the volumetric flow rate measured by the LDV instrument:

\[
Q_v = \frac{v_{\text{axis}} \pi d^2}{4 \left( a + b \ln \left( \frac{\rho \mu \pi d}{\rho} \right) \right) + \epsilon}. \tag{A.2}
\]

The symbols are listed in table A1.

### A.1.1. Axial velocity

Generally speaking, the velocity \( v \) of a fluid particle in the flow measured in LDV is determined from

\[
v = \frac{I (f_D - f_S)}{\cos(\gamma)} \tag{A.3}
\]

where \( I \) is the fringe spacing, \( f_D \) is the Doppler frequency measured by the system as the particle flows through the measuring volume, \( f_S \) is a frequency shift deliberately added to one of the beams in order to retrieve directional information and, \( \gamma \) is the offset from 90° at the measurement spot.

The factor \( 1 / \cos(\gamma) \) takes into account a possible misalignment between the optical axis and the particle velocity. The angle \( \gamma \) does not merely depend on a careful alignment of the LDV system with respect to the windows in the pipeline but is also affected by small imperfections in the window mounts. If the front and back windows are skewed, the laser beams are slightly deflected and the fringes in the measurement volume are aligned along the bisection line between converging beams. For small angles \( \alpha \) between the windows, the resulting misalignment with the flow velocity is approximately \( \gamma \approx \alpha/2 \).

In the plane wave approximation, the fringe spacing is only determined by the wavelength of the laser beam and the beam angle. The geometry also determines the size of the measuring volume via the beam angle and the beam width. However, the laser beams are better modelled as Gaussian beams (see Mickan et al [4] for a discussion pertaining to LDV, and Brooker [12] for more on Gaussian beams in general). Depending on whether the beam waists are mapped exactly to the focal spot, the fringe spacing may change in the measurement volume.

The physical model in equation (A.3) is used to construct a more elaborate measuring function. In normal operation, the fringe spacing is an input to the system and the output available to the operator is the deduced velocity. The measurement function is therefore re-phrased in terms of a velocity, and a small correction is added to the value. This correction is linearized around the measured velocity value:

\[
v_{\text{axis}} = v_{\text{obs}} + r + \frac{\partial v}{\partial f_D} f_{D,\text{corr}} + \frac{\partial v}{\partial f_S} f_{S,\text{corr}} + \frac{\partial v}{\partial I_{\text{cal}}} I_{\text{cal}} + \frac{\partial v}{\partial \gamma} \gamma_{\text{corr}}. \tag{A.4}
\]

The sensitivity coefficients are computed from equation (A.4), and expressed in terms of the calibrated value for the fringe spacing \( I \) and observed velocity \( v_{\text{obs}} \). Table 2 presents an overview of symbols used in the measurement function for the axial velocity.

In general, a measurand \( Y \) is determined from \( N \) other input quantities \( X_i \) through a functional relationship \( f \):

\[
Y = f (X_1, X_2, \ldots, X_N). \tag{A.5}
\]

The ‘Guide to the Expression of Uncertainty in Measurement’ (GUM [13]) provides a framework for evaluating the uncertainty associated with measurements. According to the GUM, the standard uncertainty \( u_c(y) \) of the estimate \( y \) of \( Y \) obtained from the input estimates \( x_i \) of input quantities \( X_i \) is given in first approximation by the root square of the combined variance \( u_c(y)^2 \), itself given by

\[
u_c^2 (y) = \sum_{i=1}^{N} c_i^2 u^2 (x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u (x_i) u (x_j) r(x_i, x_j) \tag{A.6}
\]
Table A2. Overview of symbols used in the measurement function for the axial velocity.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{\text{obs}})</td>
<td>Observed velocities using the LDV system</td>
<td></td>
</tr>
<tr>
<td>(r)</td>
<td>Resolution of read-outs</td>
<td></td>
</tr>
<tr>
<td>(f_{D,\text{corr}})</td>
<td>Correction for Doppler frequency</td>
<td>best estimate is 0</td>
</tr>
<tr>
<td>(f_{S,\text{corr}})</td>
<td>Correction for acousto-optical frequency shift</td>
<td>best estimate is 0</td>
</tr>
<tr>
<td>(I_{\text{cal}})</td>
<td>Fringe spacing calibration correction</td>
<td>best estimate is 0</td>
</tr>
<tr>
<td>(\gamma_{\text{corr}})</td>
<td>Misalignment between optical axis and flow axis</td>
<td>best estimate is 0</td>
</tr>
</tbody>
</table>

Sensitivity coefficients

\(c_{f_0} = \frac{\partial f}{\partial x_0} = 1\)  
Carrier frequency correction

\(c_{f_0} = \frac{\partial f}{\partial x_0} = -1\)  
Frequency shift correction

\(c_{f_0} = \frac{\partial f}{\partial x_0} = \frac{\langle u_0 \rangle}{r}\)  
Fringe spacing

\(c_{f_0} = \frac{\partial f}{\partial x_0} = f(\langle u_0 \rangle) (v_{\text{obs}})\)  
Misalignment between optical and flow axes

Table A3. Sensitivity coefficients used in the measurement function for volumetric flow rate.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_d)</td>
<td>Sensitivity with throat diameter</td>
<td></td>
</tr>
<tr>
<td>(c_{v_{\text{axis}}})</td>
<td>Sensitivity with axial velocity</td>
<td></td>
</tr>
<tr>
<td>(c_{A^*})</td>
<td>Sensitivity with model function coefficient</td>
<td></td>
</tr>
</tbody>
</table>

where,

- \(c_i = \frac{\partial f}{\partial x_i}\) is the sensitivity coefficient associated with estimate \(x_i\) of input quantity \(X_i\),
- \(u_i(x_i)\) is the standard uncertainty of estimate \(x_i\),
- \(\rho(x_i, x_j)\) is the cross-correlation coefficient between the uncertainty in \(x_i\) and \(x_j\).

The quantity \(u_i(y)\) is the combined standard uncertainty of the measurement result \(y\) and the quantity \(U_i(y) = k * u_i(y)\) is the expanded uncertainty. \(k\) is the coverage factor chosen based on the level of confidence required of the interval \((y - U_i(y), y + U_i(y))\).

A.1.2. Volumetric flow rate  
The sources of uncertainty considered in assessing the combined standard uncertainty of the LNG volumetric flow rate are Type A standard uncertainty

- Axial velocity: estimated from the experimental standard deviation determined from repeated measurements under the best flow conditions, fringe spacing, Doppler frequency and alignment of the optical axis

Type B standard uncertainty

- Internal diameter \(d\) of the LDV device convergent throat: this component is obtained by direct measurement (see Mickan et al [4]) and corrected for a systematic effect of temperature

- Uncertainty on the \(a\) constant of the model function: this component is estimated from Monte Carlo simulation based on the calibration measurements performed in air and liquid nitrogen (LN2)

- Uncertainty on the \(b\) constant of the model function: this component is estimated from Monte Carlo simulation based on the calibration measurements performed in air and LN2

- Fluid density: calculated from composition, temperature and pressure measurement

- Fluid viscosity: calculated from composition, temperature and pressure

- Other: uncertainty contribution due to choose of model function, term \(\epsilon\)

Equation (3) is re-written as

\[
Q_x = \frac{v_{\text{axis}} \pi d^2}{4A^*}.
\]  
(A.7)

\(A^*\) can be seen as a scaling factor derived from the model function and relates axial and cross-sectional average velocity for the particular geometry of the LDV instrument.

Table A3 gives an overview of the sensitivity coefficients obtained for the expression [14] of the volumetric flow rate:

Separate uncertainty budgets for input quantities \(d\), \(v_{\text{axis}}\) and \(A^*\) are required to calculate the uncertainties for these input quantities. In addition \(d\), \(v_{\text{axis}}\) and \(A\) are not independent variables and a correlation matrix must be determined.

In this article and for illustration purposes, the uncertainty budget will be presented for a randomly-chosen measurement with liquid nitrogen at NIST in laboratory conditions in table A4 and the correlation matrix in table A5.

A.1.3. Axial velocity  
The measurement function is equation (A.6) and the sensitivity coefficients are listed in table A1. With the nomenclature from the table, equation (A.8) becomes

\[
u_i^2 = \nu_{\text{obs}}^2 + \nu_t^2 + \nu_{fr}^2 \nu_{fr}^2 + \nu_{fr}^2 \nu_{fr}^2 + \nu_{fr}^2 + c^2 u_i^2 + c^2 u_i^2 + 2c u_i u_{\text{obs}}.
\]  
(A.8)

The final term is a correlation term between the fringe spacing calibration uncertainty and the scatter in the observed velocity. The correlation coefficient is assigned the value 1 because properties of the LDV system dominate both, which will be explained in greater detail below.

The uncertainty in the frequency determination is assumed to be small. We also note that, strictly speaking, traceable frequencies are not necessary here: the traceability is via the fringe spacing and the output velocities of the system.

A.1.4. Misalignment  
A standard analytical approach for the uncertainty due to misalignment results in a sensitivity coefficient \(c_{\gamma} = 0\). A proper evaluation requires a Monte Carlo analysis of the contribution.

Equation (A.8) was used to model the relative contribution of uncertainty in \(\gamma\). The equation may be rewritten in terms of
a reference velocity $v_{\text{ref}}$ measured if there is no misalignment: $v_{\text{obs}} = v_{\text{ref}}/\cos(\gamma)$. Values for $\gamma$ were drawn from a normal distribution with mean value 0 and standard deviation $\sigma_{\gamma}$, and the standard deviation of output velocity $u_{v,\text{obs}}$ recorded. The relative scatter in velocity, $f(\gamma) = u_{v,\text{obs}}/v_{\text{ref}}$ can be used as a quantification of the relative uncertainty contribution of the angular misalignment.

After repeating the calculation for a range of different $\sigma_{\gamma}$ we obtain the curve shown in figure A1. The curve may be used to find the value of $f(\gamma)$ for a given misalignment uncertainty, and the contribution to uncertainty in velocity is found by multiplying $f(\gamma)$ and $\langle v_{\text{obs}} \rangle$.

A.1.5. Repeated measurements The LDV technique has two main sources of scatter in the output: (i) the fringe spacing depends on the location in the measurement volume, and (ii) a wide scatter in the measured frequency (and hence in the deduced velocity), illustrated in figure A2. It shows a histogram generated from almost 70 000 burst recordings acquired...
in roughly 20s. The distribution resembles a normal distribution with a standard deviation of roughly 0.027 m s⁻¹ (1% of the measured velocity in this case).

The scatter is significant. It could be caused by fluctuations in the reference velocity, or mechanical vibrations that affect the distance between the reference source and the LDV system. Intrinsic properties of LDV system could also play a role, such as the signal processing. When the system is applied to flow measurements additional scatter may be caused by a poorer signal quality; both the number of particles in the measurement volume during the burst, and the time that particles are present in the measurement volume during the burst, can affect the signal quality.

The fringe spacing variation depends on the LDV system (objective lens, wavelength etc). It is probably caused by the Gaussian nature of the laser beams if the beam waist is located outside the focal point of the lens. In well-aligned and well-designed systems, it may be almost constant, but it will typically be around 1% in practical designs.

The distribution of observed velocities for particles moving through the measurement volume in arbitrary locations is the result of both sources of scatter. The exact shape of the output distribution will depend on the fringe spacing variation, but the variance of the distribution can be obtained approximately from the sum of the variances of the histogram in figure A2 and a rectangular distribution with edges given by the extremes of the fringe spacing. The mean of the distribution corresponds to the velocity of a particle moving through the centre of the measurement volume, triggering a frequency response corresponding to the mean value of the intrinsic distribution.

The effect of repeated sampling is then to sample both the measurement volume and the intrinsic scatter of the LDV device simultaneously. We define the measurand as the velocity at the centre of the measurement volume; repeated sampling may be employed to estimate this measurand as the average of multiple particles will converge on the centre value.

As an example, here is a case where the observed velocity is 2.04 m s⁻¹ (figure A3). The fringe spacing and associated uncertainty is taken from a calibration certificate from December 2016, using the 160 mm objective lens. The input values are summarised as follows:
The expanded uncertainty (95% confidence interval) \( U_c \) is equal to 0.0068 m s⁻¹ (0.33% relative uncertainty). The uncertainty in the fringe spacing calibration dominates the uncertainty budget (73%).

The contribution from the fringe spacing calibration is treated in an unusual manner. A calibration uncertainty should usually be treated as a type B uncertainty. However, the uncertainty stated in the calibration certificate is dominated by the variation inside the measurement volume. This is appropriate for a single measurement of an individual particle, but the average of multiple particles will converge on the centre value. The calibration uncertainty is therefore treated as a type A uncertainty in this case, but with the caveat that it may never fall below the uncertainty of the reference velocity used in the calibration experiment.

### Table A6. Sensitivity coefficients used in the measurement function for throat diameter.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_d )</td>
<td>( \frac{\partial \bar{d}}{\partial \bar{d}} = 1 + \alpha (T - T_i) )</td>
<td>Sensitivity with throat diameter at reference temperature</td>
</tr>
<tr>
<td>( c_\alpha )</td>
<td>( \frac{\partial \bar{d}}{\partial \bar{\alpha}} = d_i (T - T_i) )</td>
<td>Sensitivity with thermal expansion coefficient</td>
</tr>
<tr>
<td>( c_T )</td>
<td>( \frac{\partial \bar{d}}{\partial \bar{T}} = d_i \alpha )</td>
<td>Sensitivity with ambient temperature</td>
</tr>
<tr>
<td>( c_{T_i} )</td>
<td>( \frac{\partial \bar{d}}{\partial \bar{T_i}} = -d_i \alpha )</td>
<td>Sensitivity with reference temperature</td>
</tr>
</tbody>
</table>

#### A.16. Throat diameter

The throat diameter of the convergent is measured by a calibration laboratory at 20 °C. To obtain the throat diameter at the local temperature, a temperature correction is applied based on the coefficient of thermal expansion of the LDV instrument material:

\[
d = d_i \cdot [1 + \alpha (T - T_i)]
\]

(A.9)

where

- \( d_i \) is the diameter of the throat at 20 °C
- \( \alpha \) is the coefficient of thermal expansion of AISI 316 steel (K⁻¹)
- \( T \) is the temperature at measurement conditions (K)
- \( T_i \) is the reference temperature at throat diameter measurement, here 20 °C (K)

Table A6 gives an overview of sensitivity coefficients obtained for this expression of the throat diameter:

Table A7 lists the sources and values of the uncertainties for the throat diameter for the chosen example. For this particular measurement, table A7 indicates that the throat diameter is 0.039856 m with an expanded uncertainty (95% confidence interval) \( U_c \) equal to 0.000030 m (0.076% relative uncertainty).

The throat diameter is measured by an accredited laboratory and values are reported at 20 °C. An inside circle of diameter 399.87 μm with a cylindricity default of 19 μm is reported. In the uncertainty budget, a mean diameter at 20 °C of 399.87 ± 19/2 = 399.965 μm is considered for the throat with a standard uncertainty of 0.014% relative uncertainty.

The thermal expansion coefficient (TEC) for convergent material AISI 316 is stated in data sheets to be 16.5 ppm K⁻¹ in the range 20–100 °C. The TEC depends on temperature and a study by Thermeau [11] shows that the TEC decreases for steel when temperature reaches cryogenic temperatures (0–150K). Thermeau in [11] (for a similar material (AISI 304L)) is used to derive a maximum difference between data sheet values and TEC at low temperatures in the order of 2.5 ppm K⁻¹. In the uncertainty budget, a standard relative uncertainty of 10% is assumed for the TEC (see [14]).

The LDV instrument has no temperature sensor and the local fluid temperature \( T \) is measured at locations upstream or downstream of the LDV instrument. The uncertainty of this measurement will depend on the installation where the measurement is performed and is expected to be lower under...
Sensitivity with model function

\[ \frac{\partial \epsilon}{\partial A} = 1 \]

Sensitivity with natural logarithm of 293 K

\[ \frac{\partial \epsilon}{\partial \ln T} = 1 \]

Sensitivity with linear regression

\[ \frac{\partial \epsilon}{\partial b} = 1 \]

\[ \frac{\partial \epsilon}{\partial \alpha} = X \]

\[ \frac{\partial \epsilon}{\partial X} = b \]

Laboratory conditions and higher under field conditions. In addition, some unknown temperature drop can occur if the temperature measurement is carried out far from the LDV instrument. For this uncertainty budget, a conservative (high) estimate of the standard uncertainty of 1 K is assumed.

The throat diameter calibration certificate states that there is an uncertainty in the reference temperature under measurements of 1 K (coverage factor not stated). This temperature is an uncertainty in the reference temperature under measurement has negligible impact on the overall uncertainty budget. For the chosen measurement, a conservative (high) estimate of the standard uncertainty of 1 K is assumed.

In these experiments, flow measurements are conducted where the LDV instrument and the facilities’ respective flow standards are mounted in series. Once the flow has stabilized, the axial velocity \( v_{\text{axis}} \) is measured by the LDV instrument while the volumetric flow rate \( Q_{\text{v,standard}} \) is measured by the primary standard. The cross-sectional average velocity \( \bar{v} \) is obtained from

\[ \bar{v} = \frac{Q_{\text{v,standard}}}{\pi d} \]  

(A.11)

Table A7. Uncertainty budget for throat diameter.

<table>
<thead>
<tr>
<th>( X_i )</th>
<th>( x_i )</th>
<th>Unit</th>
<th>( u(x_i) )</th>
<th>( c_i )</th>
<th>( u(y) )</th>
<th>Contrib (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_t )</td>
<td>0.039997</td>
<td>m</td>
<td>( 5.48 \times 10^{-6} )</td>
<td>9.96 \times 10^{-1}</td>
<td>0.000005</td>
<td>13</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 1.65 \times 10^{-5} )</td>
<td>1/K</td>
<td>( 1.7 \times 10^{-6} )</td>
<td>( -8.513255025 )</td>
<td>0.000014</td>
<td>87</td>
</tr>
<tr>
<td>( T )</td>
<td>80</td>
<td>K</td>
<td>1</td>
<td>6.60 \times 10^{-7}</td>
<td>0.000001</td>
<td>0</td>
</tr>
<tr>
<td>( T_i )</td>
<td>293</td>
<td>K</td>
<td>1</td>
<td>( -6.60 \times 10^{-7} )</td>
<td>0.0000007</td>
<td>0</td>
</tr>
<tr>
<td>( Y = d )</td>
<td>0.039856</td>
<td>m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A8. Sensitivity coefficients used in the measurement function for \( A^* \).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_a )</td>
<td>( \frac{\partial A}{\partial a} = 1 )</td>
<td>Sensitivity with linear regression coefficient ( a ) (intercept)</td>
</tr>
<tr>
<td>( c_b )</td>
<td>( \frac{\partial A}{\partial b} = X )</td>
<td>Sensitivity with linear regression coefficient ( b ) (slope)</td>
</tr>
<tr>
<td>( c_X )</td>
<td>( \frac{\partial A}{\partial X} = b )</td>
<td>Sensitivity with natural logarithm of Reynolds number ( X )</td>
</tr>
<tr>
<td>( c_\epsilon )</td>
<td>( \frac{\partial A}{\partial \epsilon} = 1 )</td>
<td>Sensitivity with model function error ( \epsilon )</td>
</tr>
</tbody>
</table>

A.17 Model function and scaling factor \( A^* \)

\( A^* = \frac{v_{\text{axis}}}{\bar{v}} = a + bX + \epsilon \)  

(A.10)

where,

* \( a \) is the intercept of the model function,
* \( b \) is the slope of the model function,
* \( X = \ln(\text{Re}_d) \) is the natural logarithm of the Reynolds expressed as function of throat diameter \( d \),
* \( \epsilon \) is the error resulting from an incorrect choice of regression model.

As previously discussed, we assume in this work a linear relationship between \( A^* \) and \( X = \ln(\text{Re}_d) \). The linear regression coefficients \( a \) and \( b \) are calculated based on calibration experiments performed with air and liquid nitrogen at, respectively, the Cesame and NIST facilities, 13 experiments in total.

Table A9. Uncertainty budget for model function coefficient \( A^* \).

<table>
<thead>
<tr>
<th>( X_i )</th>
<th>( x_i )</th>
<th>Unit</th>
<th>( u(x_i) )</th>
<th>( c_i )</th>
<th>( u(y) )</th>
<th>Contrib (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>1.166</td>
<td>—</td>
<td>0.00709</td>
<td>1.000</td>
<td>0.007</td>
<td>46</td>
</tr>
<tr>
<td>( b )</td>
<td>-0.012</td>
<td>—</td>
<td>0.0052</td>
<td>13.011</td>
<td>0.007</td>
<td>42</td>
</tr>
<tr>
<td>( X )</td>
<td>13.011</td>
<td>—</td>
<td>0.021</td>
<td>0.998</td>
<td>0.000</td>
<td>0</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0</td>
<td>—</td>
<td>0.0036</td>
<td>1.0000</td>
<td>0.004</td>
<td>12</td>
</tr>
<tr>
<td>( Y = A^* )</td>
<td>1.015</td>
<td>—</td>
<td></td>
<td></td>
<td>0.010</td>
<td>100</td>
</tr>
</tbody>
</table>

Table A8 gives an overview of sensitivity coefficients obtained for \( A^* \).

Table A9 lists the sources and values of the uncertainties for model function coefficient \( A^* \):

There is a correlation between regression coefficients \( a \) and \( b \) which is obtained from Monte Carlo simulation as explained in the next paragraph. For the chosen measurement, table A8 indicates that the conversion factor \( A^* \) is 1.015 with an expanded uncertainty (95% confidence interval) \( U_c \) equal to 0.008 (0.76% relative uncertainty).

A.18 Linear regression coefficients \( a \) and \( b \)

The measurements were carried out at Cesame–Exadebit and NIST and they can be seen as calibration measurements of the LDV instrument. Figure A4 shows the same data on a plot with uncertainty bars.

To determine the linear regression coefficients \( a \) and \( b \), a Monte Carlo simulation has been carried out. At each iteration, data points are initiated within a domain defined by the
measurement uncertainties along the $x$-axis ($\ln(Re_d)$) and $y$-axis ($A^*$). Normal Gaussian distributions are considered with a mean equal to the measurement value reported and a standard deviation equal to the standard uncertainty.

Output from 10,000 iterations is used to calculate an arithmetical average of the regression coefficients and its standard deviation. The correlation coefficient between $a$ and $b$ is also calculated. It has been confirmed that 10,000 iterations are sufficient to obtain convergence in these values. Figure A5 presents the results from the Monte Carlo simulation.

A relative standard uncertainty of 1% in the intercept $a$ and 5% in the slope $b$ is obtained with this method. It is expected that this uncertainty can be reduced if:

- the measurement uncertainty in $A^*$ and $\ln(Re_d)$ can be reduced
- additional measurements can be added (additional data points)
- an alternative model function that better represents the data is implemented

In this appendix, the uncertainty budget for the volumetric mass flow rate using LDV standard has been presented. The axial velocity measurement at 2.04 m s$^{-1}$ gives a 0.6% ($k = 2$) uncertainty under cryogenic conditions. This uncertainty budget can be improved by obtaining an accurate estimation of the velocity ratio dependence as a function of the Reynolds number. Cesame–Exadebit will implement a new $A^*$ in the near future.

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**References**


