Evaluation uncertainty in transferred LNG volume

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<th>unit</th>
<th>description</th>
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<td>$c_{\text{list,cal}}$</td>
<td>[°]</td>
<td>Correction related to the calibration of the inclinometer for list</td>
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<tr>
<td>$c_{\text{list,location}}$</td>
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<td>Corrections in volume. This correction should include effects of sagging and hogging, hydrostatic pressure (if not included in gauge table) and deformation of tank during time due to drift and tank modifications (subfunction)</td>
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<tr>
<td>$C_{\text{gauge,}T(T_{\text{gauge}})}$</td>
<td>-</td>
<td>Correction factor to account for temperature effect on the level gauge. Depends on the technology used (subfunction)</td>
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<tr>
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<td>-</td>
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<td>$h_{\text{ind}}$</td>
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<tr>
<td>$H_{\text{table,}list}$</td>
<td>[m]</td>
<td>List correction read from list correction table. If resolution gives rise to significant contribution to</td>
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the combined uncertainty, linearization may be applied increasing the resolution

<table>
<thead>
<tr>
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<td><strong>List</strong></td>
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<td><strong>Trim</strong></td>
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<td><strong>T(_{ref, level gauge})</strong></td>
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<td>Reference temperature for the main gauge table</td>
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<td>[*]</td>
<td>List truncated down to nearest value available in correction gauge table</td>
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<td>Trim truncated down to nearest value available in correction gauge table</td>
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<td>[m(^3)]</td>
<td>Volume correction for sagging and hogging</td>
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<td>(\Delta V / \Delta h)</td>
<td>[m(^2)]</td>
<td>The sensitivity in the volume at the indicated level. May be read/calculated from the main gauge table or obtained from a theoretical model of the tank</td>
</tr>
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<td>( \alpha )</td>
<td>[1/K]</td>
<td>Linear Expansivity of the material in the dimensional structure of the tank</td>
</tr>
<tr>
<td>( \beta )</td>
<td>[1/K]</td>
<td>Linear Expansivity of the material in the level gauge</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>[1/Pa]</td>
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1 Introduction

This study is part of the EMRP project “Metrology for Liquefied Natural Gas (LNG)” [19] and focuses on the uncertainty of static LNG volume measurements as encountered in (un)loading LNG ships. For the uncertainty in the (un)loaded LNG volume one usually refers to the LNG custody transfer handbook of G.I.I.G.N.L. [18], in which a value of 0.42% is claimed (level of confidence of 95%, see Section 15.5). However, the LNG custody transfer handbook is not a standard but a document providing guidance to the industry by describing common practice. It can, therefore, not be used as a norm or standard. In addition, a thorough uncertainty assessment is lacking in the view of the authors. For example, several uncertainty contributions have not been validated and covariance’s are not accounted for in the combination of uncertainty sources. As of yet a thorough metrologically sound uncertainty budget has not been conducted, probably because LNG shipping is typically bound by long-term contracts. Furthermore, buyers and sellers see uncertainty as inherent to level gauging.

The goal of this report is, therefore, to evaluate the uncertainty model adopted in [18] and to validate the uncertainty figure of the (un)loaded LNG volume. In order to achieve this goal various experts in the field of LNG shipping have been interviewed. In addition, a significant data set of LNG (un)loadings has been collected and analyzed. Other studies carried out dealing with the uncertainty in tank gauging are by the Southwest Research Institute and by Benito et al. [13] (in which results from [14], [15] and [16] are used). However, not all findings between Benito and the current study are in agreement.

This report does not claim to provide the final conclusions about the level of uncertainty regarding LNG (un)loadings. The scope of this report is more to evaluate the existing model and to recommend further detailed study of specific (and sometimes new) elements. Note, the models and uncertainty budgets developed in this report can be generally applied to Moss and Membrane type tanks. However, the magnitude of the uncertainty sources should be determined specifically for each ship.

The main conclusions of this report are: 1) the uncertainty in LNG (un)loaded as stated in [1] is not evaluated/ propagated for a (un)loading situation in a metrologically sound manner and 2) the uncertainty in LNG (un)loaded as stated in [18] is optimistic (chapter 3.2). This is in contrast with what is commonly believed, i.e. that the uncertainty achieved in LNG volumes is typically in line with the GIIGNL [8].
2 Uncertainty budget

The statement of the result of a measurement is complete only if it contains both the value attributed to the measurand and the uncertainty of measurement associated with that value.

In this report all quantities which are not exactly known are treated as random variables, including the influence quantities which may affect the measured value.

The uncertainty of measurement is a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand. In this document the shorthand term uncertainty is used for uncertainty of measurement if there is no risk of misunderstanding.

The uncertainty budgets in this report are established according to Evaluation of measurement data - Guide to the expression of uncertainty in measurement (GUM). JCGM 100:2008. The methods used when establishing an uncertainty budget are not described and discussed in details in this report. Please refer to GUM for more information. The GUM is available online at: Available online at: http://www.bipm.org/en/publications/guides/gum.html.

2.1 Glossary of terms

This section gives an explanation to the uncertainty terms used in the report.

Arithmetic mean
The sum of values divided by the number of values.

Best measurement capability
The smallest uncertainty of measurement that a laboratory can achieve within its scope of accreditation, when performing more or less routine calibrations of nearly ideal measurement standards intended to define, realise, conserve or reproduce a unit of that quantity or one or more of its values, or when performing more or less routine calibrations of nearly ideal measuring instruments designed for the measurement of that quantity.

Correlation
The relationship between two or several random variables within a distribution of two or more random variables.
Correlation coefficient
The measure of the relative mutual dependence of two random variables, equal to the ratio of their covariance to the positive square root of the product of their variances.

Covariance
The measure of the mutual dependence of two random variables, equal to the expectation of the product of the deviations of two random variables from their respective expectations.

Coverage factor
A numerical factor used as a multiplier of the standard uncertainty of measurement in order to obtain an expanded uncertainty of measurement.

Coverage probability
The fraction, usually large, of the distribution of values that as a result of a measurement could reasonably be attributed to the measurand.

Experimental standard deviation
The positive square root of the experimental variance.

Expanded uncertainty
A quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

Experimental variance
The quantity characterising the dispersion of the results of a series of \( n \) observations of the same measurand given by equation (3.2) in the text.

Input estimate
The estimate of an input quantity used in the evaluation of the result of a measurement.

Input quantity
A quantity on which the measurand depends, taken into account in the process of evaluating the result of a measurement.

Intrinsic uncertainty
The uncertainty of the measurement equipment itself.

Measurand
The particular quantity subject to measurement.

Output estimate
The result of a measurement calculated from the input estimates by the model function.
Output quantity
The quantity that represents the measurand in the evaluation of a measurement.

Pooled estimate of variance
An estimate of the experimental variance obtained from long series of observations of the same measurand in well-characterized measurements under statistical control.

Probability distribution
A function giving the probability that a random variable takes any given value or belongs to a given set of values.

Quadratic summation
The square root of the summation of all elements squared.

Random variable
A variable that may take any of the values of a specified set of values and with which is associated a probability distribution.

Relative standard uncertainty of measurement
The standard uncertainty of a quantity divided by the estimate of that quantity.

Repeatability
Closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement.

Reproducibility
Closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement.

Sensitivity coefficient associated with an input estimate
The differential change in the output estimate generated by a differential change in an input estimate divided by the change in that input estimate.

Standard deviation
The positive square root of the variance of a random variable.

Standard uncertainty of measurement
The uncertainty of measurement expressed as the standard deviation.

Type A evaluation method
The method of evaluation of uncertainty of measurement by the statistical analysis of series of observations.
Type B evaluation method
The method of evaluation of uncertainty of measurement by means other than the statistical analysis of series of observations.

Uncertainty budget
Budget that contains all uncertainty sources including the distribution and coverage factor. The end result of the uncertainty budget is the total uncertainty in the parameter of interest (in this case the total LNG transferred).

Uncertainty of measurement
A parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

Variance
The expectation of the square of the deviation of a random variable about its expectation.

2.2 Mathematical model

In this section the mathematical model is discussed that will be used to propagate the uncertainties to obtain the total uncertainty in the LNG volume transferred. This model corresponds to how typically the (un)loaded volume of LNG is determined, see Figure 1 Model for (un)loading LNG. As several significant contributions are influenced by the type of tank, this is acknowledged by separation of text in this document. The mathematical model is used to determine the impact of the uncertainties on the total LNG volume (un)loaded. It is both valid for a Membrane tank (also called an integrated or depended tank) and a Moss tank (also called a independent tank or a self-supporting tank).

2.2.1 Volume

The volume in a tank \( V_{\text{table}} \) from the main gauge tables is determined with:

\[
V_{\text{table}} = V_{\text{TABLE}}(\text{Trunc}(h)) + \frac{\Delta V_{\text{TABLE}}}{\Delta h}(h - \text{Trunc}(h)), \tag{1}
\]

where \( V_{\text{TABLE}} \) is the tank table, \( \text{Trunc}(h) \) is the corrected level truncated to the nearest value available in the tank table and \( \frac{\Delta V}{\Delta h} \) is the sensitivity of the volume to the level gauge. The second term between brackets of Equation 1 is only relevant when the level gauge is in that part of the tank table with relatively large intervals, say centimeters rather than millimeters. Next, the volume obtained is corrected as follows:
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Figure 1 Model for (un)loading LNG.

\[ V = (V_{\text{table}} + c_Y) C_{\text{tank}, T} C_{\text{tank}, p}(p), \]  
\[ \text{Eq. 2} \]

where \( c_Y \) is a correction for, amongst others, sagging and hogging, hydrostatic pressure (when not included in tank table), tank modifications not accounted for and drift. \( C_{\text{tank}, T}(T) \) and \( C_{\text{tank}, p}(p) \) are correction terms for, respectively, the temperature and pressure. Note, in case of Membrane tanks, the correction term for temperature is for the ambient temperature.

2.2.1.1 Volume correction for pressure

In [8] a correction factor for the pressure is given by:

\[ c_{\text{tank}, p}(p) = 1 + \beta (p_{\text{tank}} - p_{\text{ref}}), \]  
\[ \text{Eq. 3} \]
where $\beta$ is the mean coefficient of volumetric expansion, $p_{\text{ref}}$ is the reference pressure for which the tank table is valid and $p_{\text{tank}}$ is the operating pressure. Note, because the pressure in the tanks is approximately equal to ambient and reference pressure, the volume correction for pressure is usually neglected.

### 2.2.1.2 Volume correction for temperature

A common correction factor for the temperature deviation from reference conditions is [8]:

$$c_{\text{tank},T}(T) = 1 + 3\alpha(T_{\text{tank}} - T_{\text{ref}}), \quad \text{Eq. 4}$$

where, $\alpha$ is the mean coefficient of linear thermal expansion of the tank material, $T_{\text{ref}}$ is the reference temperature for which the tank table is valid and $T$ is the average tank temperature. Although the above equation is commonly used, it is not entirely correct. The correct formula for taking into account the volume correction for temperature is:

$$c_{\text{tank},T}(T) = \left(1 + \alpha(T_{\text{tank}} - T_{\text{ref}})\right)^3, \quad \text{Eq. 5}$$

Notes:
- The volume correction for temperature is especially relevant for Moss tanks [18]. This is because for a Moss tanks the supporting structure cools down to LNG temperatures, whereas for Membrane tanks the temperature of the supporting structure is approximately equal to ambient (sea water) temperature.
- The volume correction for sea water temperature is only relevant for membrane type tanks as the supporting structure is outside the isolation. As of yet the influence of the sea water temperature is not taken into account [18].

### 2.2.1.3 Other volume corrections

In Equation 2 there is also a term given for other volume corrections ($c_V$). Amongst others this term includes a volume change due to:
- sagging and hogging ($\Delta V_{\text{sagging\&hogging}}$)
- hydrostatic pressure when not included in tank table ($\Delta V_{\text{Hydrostatic}}$)
- not accounted for tank modifications and drift ($\Delta V_{\text{Drift}}$)

This term can therefore be written as:

$$c_V = \Delta V_{\text{sagging\&hogging}} + \Delta V_{\text{Hydrostatic}} + \Delta V_{\text{Drift}}. \quad \text{Eq. 6}$$

Unfortunately, analytical expressions are not available for these type of corrections because of their complexity or because they are unknown,
hence the uncertainty. The components given above, therefore, have a value zero and a finite uncertainty.

### 2.2.2 Level gauge

The level is determined by:

\[
h = h_{\text{ind}} \cdot C_{\text{gauge, } T}(T_{\text{gauge}}) \cdot C_{\text{gauge, } p}(p_{\text{gauge}}) + \Delta h_{\text{trim}} + \Delta h_{\text{list}} + \Delta h_p + \Delta h_{\text{comp}} + \Delta h_{\text{cal}} + \Delta h_{\text{drift}},
\]

\[\text{Eq. 7}\]

where:

- \( h \) corrected level gauge
- \( h_{\text{ind}} \) observed/ indicated level gauge
- \( C(T_{\text{gauge, } \text{mean}}) \) correction for temperature (vapor and/ or liquid)
- \( C(p_{\text{gauge}}) \) correction for pressure effect on level gauge and position level gauge
- \( \Delta h_{\text{trim}} \) correction for nonzero trim
- \( \Delta h_{\text{list}} \) correction for nonzero list
- \( \Delta h_p \) correction for a density deviation from reference conditions
- \( \Delta h_{\text{comp}} \) a correction for a composition deviation from reference conditions
- \( \Delta h_{\text{cal}} \) correction according the calibration certificate
- \( \Delta h_{\text{drift}} \) correction for drift of level gauge

The formulation for the level is a general one and can be applied to any type of level gauging. However, the various contributions are not relevant for all level types. For example, the correction for density is mainly relevant for a float type level gauge. Furthermore, depending on the definition of the correction for temperature \( C(T_{\text{gauge, } \text{mean}}) \) and pressure \( C(p_{\text{gauge}}) \), these corrections can also be added to the observed level gauge. The correction for trim is defined as:

\[
\Delta h_{\text{Trim}} = H_{\text{TABLE}}(\text{Trunc}(\text{Trim})) + \frac{\Delta H_{\text{TABLE}}}{\Delta \text{Trim}}(\text{Trim} - \text{Trunc}(\text{Trim})), \quad \text{Eq. 8}
\]

with:

\[
\text{Trim} = \text{Trim}_{\text{ind}} + c_{\text{trim, location}} + c_{\text{trim, cal}}, \quad \text{Eq. 9}
\]

Where:

- \( H_{\text{TABLE}} \) correction table for trim
- \( \text{Trunc}(\text{Trim}) \) corrected trim truncated to the nearest value in the trim correction table
- \( c_{\text{trim, location}} \) correction for a misplacement of bending effects of the inclinometer
- \( c_{\text{trim, cal}} \) correction for calibration and drift
sensitivity of the height with the observed trim

The correction for list is similar to the correction of trim. The correction for pressure is neglected as pressure has only a marginal influence on the level gauge and the tank pressure is typically close to 1 bar. The correction required according to the calibration certificate does not require a model, it is simply a value that follows from the certificate. This is also true for drift, in case a successive calibration of the level gauge equipment indicates a constant drift, one could add a time dependent correction term to the level gauge based on the series of calibration certificates. The other corrections are next discussed in more detail.

2.2.2.1 Level correction for temperature (vapor and/or liquid)

The correction for temperature depends greatly on the type of level gauge used. For a float type level gauge the correction for temperature is given as [6]:

\[
C_{\text{gauge, } T} = (\beta \cdot (T_{\text{gauge}} - T_{\text{ref, gauge}}) + \alpha \cdot (T_{\text{tank}} - T_{\text{ref, tank}})) (H - h + \alpha T - T_0 H),
\]

where:

- \( \alpha \) volumetric expansion coefficient of the tank
- \( \beta \) linear expansion coefficient of the level gauge
- \( T_{\text{gauge}} \) the mean temperature of the level gauge. This temperature is assumed to be equal to the mean vapor temperature
- \( T_{\text{ref, level gauge}} \) reference temperature for the level gauge
- \( T_{\text{tank mean}} \) the mean temperature of the dimensional structure of the tank. This temperature is assumed to be equal to the average of the vapor and liquid temperature
- \( T_{\text{ref, tank}} \) The reference temperature of the main gauge table
- \( H \) Total length of the level gauge
- \( T \) reference temperature on which the tank table was calibrated
- \( T_0 \) temperature of the tank at the time of calibration

Note, the above equation is also applicable to other float types, however the thermal shrinkage of the level gauge is then irrelevant.

2.2.2.2 Level correction for trim and list

The correction for list and trim for a Moss type (spherical) tank is given by [6]:
\[ \Delta h_{\text{trim}} = R - (R - h) \cos \theta - h_{\text{ind}}, \]  
\text{Eq. 11}

where, \( R \) is the radius, \( \theta \) is the trim or list angle and \( h \) is defined as:

\[ h = h_{\text{ind}} - l \tan \theta, \]  
\text{Eq. 12}

where \( l \) is the longitudinal distance between the tank’s vertical axis and the level gauge.

The magnitude of the correction depends on the location of the level gauge, e.g. if the level gauge was to be positioned in the point of gravity, the correction for trim equals zero. Typically Membrane type tanks have larger corrections than other tank types because of their relative large cross section. Finally, Membrane type tanks typically have to be at nonzero trim to empty the tanks as the tank bottom is typically completely flat.

### 2.2.2.3 Level correction for density and composition

The level correction for density is given by [6]:

\[ \Delta h_p = \frac{V}{A} \left( \frac{\rho_{\text{ref}}}{\rho} - 1 \right), \]  
\text{Eq. 13}

where, \( V \) is the displacement of the float for \( \rho_{\text{ref}} \), \( A \) is the cross sectional area, \( \rho_1 \) is the reference density (used for zeroing) and \( \rho \) is the actual density. Note, the level correction for density is only relevant for float and radar type level gauges. In general the level correction for composition is rather small and therefore neglected in this study.

### 2.2.3 Volume transferred

The LNG volume transferred is simply the difference between the two volumes measured at the \( t_{\text{end}} \) and \( t_{\text{start}} \), i.e.:

\[ V_{\text{(un)loaded}} = V_{\text{end}} - V_{\text{start}}, \]  
\text{Eq. 14}

where \( V_{\text{end}} \) and \( V_{\text{start}} \) are obtained by filling in Equation 1 with the measured values at \( t_{\text{end}} \) and \( t_{\text{start}} \). This thus requires measuring all relevant physical parameters at the two time levels.

### 2.2.4 Impact of uncertainty sources on LNG volume transferred

With the functional relations between the various parameters, the LNG volume transferred can now be determined. And it is also possible to determine the sensitivity coefficients which describe the extent to which the output estimate is influenced by variations of an input estimate. In other words: The sensitivity coefficient multiplied by the magnitude of an
uncertainty source (e.g. 7 mm) influences the uncertainty of a transferred LNG volume (e.g. 0.2 %).

In mathematical terms, the sensitivity coefficient is defined as the partial derivative of the LNG transferred with respect to the uncertain parameter. This can be illustrated by the following two examples.

**Example 1: Main gauge table and volume corrections**

The sensitivity coefficients of the main gauge table and the volume corrections (with respect to the volume at start or stop) are equal to approximately 1. Consider for example the sensitivity coefficient of the volume corrections:

\[
\varepsilon_{C_V} = \frac{\partial}{\partial C_V} \left( (V_{\text{table}} + c_V) C_{\text{tank},T}(T) C_{\text{tank},p}(p) \right) = C_{\text{tank},T}(T) C_{\text{tank},p}(p) \approx 1
\]

This makes sense as the uncertainty in the main gauge tables (and volume corrections) directly influence the uncertainty in the LNG volume transferred.

**Example 2: Corrected level**

The sensitivity coefficient of the corrected level (with respect to the volume at start or stop) follows from the partial derivative of the volume with respect to the corrected level gauge, working out the equations results in:

\[
\varepsilon_{C_V} = \frac{\partial V_{\text{TABLE}}}{\partial h} C_{\text{tank},T}(T) C_{\text{tank},p}(p) \approx \frac{\partial V_{\text{TABLE}}}{\partial h}
\]

This value can be deduced from the tank tables by taking the numeric derivative of the main gauge table with respect to the corrected level gauge. Also, it can be deduced by assuming a functional relationship for the tank volume as function of the corrected level.

As mentioned earlier the influence on the uncertainty equals the sensitivity coefficient multiplied by the magnitude of the corresponding uncertainty source. Hence sensitivity coefficients equal to 0 have no impact on the overall uncertainty in transferred LNG volume, whereas a sensitivity coefficient of 1 implies that the uncertainty in the parameter of interest equals the uncertainty of the respective uncertainty source.

The uncertainty budgets for (un)loaded volume for Membrane and Moss type tanks in chapter 4 indicate that the majority of the sensitivity coefficients are equal to 1. The remainders are related to uncertainties arising from the corrected level gauge and the changes in temperature during the (un)loading process.
2.2.5 Default values of physical parameters

In order to be able to fill in the uncertainty budget it is required to have the numeric values of some parameters. These are given below:

- \( \alpha_{\text{spherical tank}} = 1.8 \cdot 10^{-5} \, ^\circ\text{C}^{-1} \) (spherical tanks are typically made from aluminum [1])

- \( \alpha_{\text{membrane tank}} = 0 \, ^\circ\text{C}^{-1} \), because:
  - isolation shrinks a bit, however this can be neglected [24]
  - membrane tanks are typically made from Invar [1]

- \( \beta = 1 \cdot 10^{-6} \, ^\circ\text{C}^{-1} \)

3 Uncertainty in the transferred amount of LNG

In this chapter the actual uncertainty in the amount of the transferred LNG is determined. This is accomplished by first quantifying the uncertain components that affect the uncertainty in the transferred LNG. Thereafter, the uncertainty budget is given for a Moss type tank (also called self-supporting or independent) and a Membrane tank (also called an integrated tank). In case the reader is not familiar with uncertainty budgets, he or she is referred to Chapter 2 for a brief explanation on the various terms.

3.1 Quantification of uncertainty sources

In this section the various components that affect the uncertainty in the transferred level of LNG are discussed and quantified. Shipping contracts commonly refer to [6]. Typically the allowed uncertainties are enclosed in the contract, e.g. uncertainty level gauging less or equal to 7.5 mm. Typical value for process parameters other than level gauging are believed to be accurate within 1%. However, it is the opinion of the authors that this uncertainty level may be optimistic. Furthermore, there is no sound metrological proof. The uncertainty sources quantified in this section are used in Sections 3.2 and 3.3 to estimate the uncertainty in LNG transferred.

Notes:
- In the literature the uncertainty budget is not always given. Consequently, it is not possible to verify which uncertainty sources have been taken into account and which are not. When this is the case, the mentioned uncertainty is taken as the intrinsic uncertainty. This may imply that some uncertainties are taken into account twice, however makes sure that the final uncertainty budget is not an underestimation.
- In case an uncertainty arises from a tolerance ($\pm T$) a uniform distribution function is assumed. This will be indicated with the following notation: $u(x) = T/\sqrt{3}$.

- It is assumed that all uncertainties given in [18] are standard uncertainties ($k=1$). This assumption is based on Section 15.5 where it is stated that the static LNG volume determination is subject to a standard uncertainty of 0.21%. However, as the coverage factor (level of confidence) in the remainder of the text is never explicitly mentioned, the assumption may not be valid for all uncertainties stated.

- As mentioned before, [18] is not a norm. It is, therefore, not entirely justified to copy the uncertainties mentioned. However, as this report aims at a reevaluation of the uncertainty given in [18], copying these uncertainties is justified.

- The uncertainty sources discussed and quantified in the following are probably not complete.

3.1.1 Main gauge table

The uncertainty in the main gauge tables is because the tables are determined with measurement equipment with finite accuracy. The uncertainty in the gauge table, $u(V_{TABLE})$, can, according to various resources, be quantifies as follows:

- $u(V_{TABLE}) = \frac{t}{\sqrt{3}}$, $t = 0.05\%$ [12]
- $u(V_{TABLE}) = 0.2\%$ [18] ($0.2\%, k = 1$)
- $u(V_{TABLE}) = 0.15\%$ [8] ($0.3\%, k = 2$)
- $u(V_{TABLE}) = \frac{t}{\sqrt{3}}$, $t = 0.05 - 0.1\%$ [9]

3.1.2 Volume corrections

3.1.2.1 Sagging and hogging for a Membrane type tank

The uncertainty due to sagging and hogging of a membrane tank ($u(\Delta V_{Sag\&Hog})$) is because sagging and hogging influences the tank and changes over time. According to a study by [15] the uncertainty due to sagging and hogging can be as much as 0.1% of the tank volume (coverage factor 1). Note, in case the LNG transfer is carried out in a well protected area, one could argue that this uncertainty can be omitted. However, in order not to underestimate the final uncertainty, this term is included in the uncertainty budget.

3.1.2.2 Hydrostatic pressure

In case the deformation due hydrostatic pressure is not included in the tank table, or based on empirical models, the tank deformation changes during transferring the LNG. This implies an uncertainty in the volume for certain level gauge, however also the uncertainty in level gauge may
increase as the position of the level gauge may change due to the tank deformation [21].

For Moss tanks the deformation is relatively easily to predict with finite element methods [23]. However, for Membrane tanks the deformation is more difficult to predict which can result in a significant uncertainty. According to a study by [15] the uncertainty due to hydrostatic pressure for a Membrane tank, can be as much as 0.1% of the tank volume.

**3.1.2.3 Calibration and drift**

The uncertainty due to drift ($u(\Delta V_{drift})$) is caused by slow deformation of the tank (creep-deformation, damages, unaccounted for modifications etc [22]). This type of uncertainty can be quantified with historical date of calibrations and recalibrations. Historical information is in general not available because there is no real interest from the industry. Furthermore, in case tank tables are remade, which happens rarely, typically the location and/or the party is different which makes comparison quite difficult.

According to [23], however, the uncertainty due to drift is negligible for non-cryogenic shipment (successive tank capacity determinations over a period of 30 years showed hardly a deviation). However, as LNG shipment implies drastic temperature changes, this may not be true of LNG shipping. For now a relative uncertainty of 0.05% of the LNG volume is assumed.

**3.1.3 Indicated level gauge**

In the below subsections the measurement/ intrinsic uncertainties are given for the most common types of level gauging. In addition to the intrinsic/ measurement uncertainty, one should also take the type A uncertainty into account. The type A uncertainty is the deviation of the mean. Loosely speaking one could say that the type A uncertainty gives information on the stability of the measurements. For example, when the LNG carrier exhibits ship movement due to waves, the series of measurement may have a significant spread, indicating a less reliable measurement.

In order to study whether the type A uncertainty is important, real shipment data has been studied. This shipment data concerns static LNG volume measurements of full and empty tanks. The primary level gauge is always of the radar type, the secondary level gauge is either of the radar type or of the float type. As waves are believed to have a major impact on the stability (standard deviation of the mean), a distinction has been made in the protection level of the LNG terminal. The following division in levels is used:

- Level 1, minor protection, e.g. open ocean with only concrete wave breaking;
- Level 2, modest protection, e.g. adjacent to open ocean with concrete wave breaking;
- Level 3, high protection, e.g. a basin.

In Table 1 the number of available data points is given. For the data points given in Table 1 the average of the standard deviation of the mean for the primary level gauge are given in Table 2. The standard deviation of the mean is computed per data point for the various level gauges (typically 5). These values are averaged per category as shown in Table 2. From this table it follows that the average standard deviation is significant larger for level gauging of full tanks than of empty tanks. This could be expected because level gauging for full tanks simply yields larger numbers and thus larger deviations etc.

**Table 1** Number of data points of shipment data. A division is made based on the secondary level gauge (radar or float), whether or not the tank is full and the protection level. Zero values for the level gauge have been discarded.

<table>
<thead>
<tr>
<th></th>
<th>radar – radar</th>
<th>radar – float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>prot lev 1</td>
<td>prot lev 2</td>
</tr>
<tr>
<td>empty</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>full</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

From Table 2 it can be deduced that the average standard deviation decreases with the protection level. This indicates that waves have a significant impact on the standard deviation of the mean and thus on the uncertainty in the LNG volume transferred. For the LNG terminals with a low level of protection, the standard deviation of the mean is significant and should be included in the uncertainty budget.

**Table 2** Average standard deviation of the mean of the primary level gauge. Values are in millimeters.

<table>
<thead>
<tr>
<th></th>
<th>prot lev 1</th>
<th>prot lev 2</th>
<th>prot lev 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>empty</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>full</td>
<td>20</td>
<td>17</td>
<td>4</td>
</tr>
</tbody>
</table>

Finally, one could also include the uncertainty source that relates to the difference between the primary and secondary level gauge. In case there is a significant difference between the primary and secondary level gauge, the measurement appears less reliable. In order to investigate whether or not this is an important parameter, the same shipment data has been studied, the result is shown in Table 3. From this table it follows that the differences in corrected levels are roughly of the same order of magnitude for empty and full tanks. Furthermore, it follows that the difference in corrected level is largest of the lowest protection level; however there is no distinct trend. Again, for the LNG terminals with a low level of protection, the difference in corrected level is important and should be included in the uncertainty budget.
Table 3 Difference in corrected level gauge between the primary and secondary level gauge. Zero values for the level gauge have been discarded. Values are in millimeters.

<table>
<thead>
<tr>
<th></th>
<th>radar – radar</th>
<th>radar – float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>prot lev 1</td>
<td>prot lev 2</td>
</tr>
<tr>
<td>empty</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>full</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

3.1.3.1 Radar type

The intrinsic/ measurement uncertainty in the indicated level gauge, $u(h_{ind})$, can be quantified as:
- $u(h_{ind}) = 3.8 \text{ mm} \ [8]$
- $u(h_{ind}) = 5 \text{ mm} \ [18]$

In case the type A uncertainty is only based on the standard deviation of the mean, the worst case scenario yields an uncertainty of:
- $u(h_{ind}) = 20 \text{ mm} \ (\text{full, type A uncertainty, see Table 2})$
- $u(h_{ind}) = 2 \text{ mm} \ (\text{empty, type A uncertainty, see Table 2})$

In case also the difference between the radar and another level gauge (float) is taken into account, the uncertainty would be much higher. Although two measurement principles should be consistent, it is perhaps too strict to include the worst case scenario (143 mm) fully as uncertainty. This requires more investigation into the different principles and more data points. However, the difference in indicated values between two the same measurement principles can be used as an additional source of uncertainty, i.e.:
- $u(h_{ind}) = 18 \text{ mm} \ (\text{full, type A uncertainty, see Table 3})$
- $u(h_{ind}) = 7 \text{ mm} \ (\text{empty, type A uncertainty, see Table 3})$

When we assume no correlation between the three types of uncertainty, the total uncertainty can be added quadratically. Taking only the worst case scenarios into account, this yields:
- $u(h_{ind}) = 27 \text{ mm} \ (\text{full})$
- $u(h_{ind}) = 9 \text{ mm} \ (\text{empty})$

In case the difference between the level gauges is included in the uncertainty the following figures are obtained:
- $u(h_{ind}) = 145 \text{ mm} \ (\text{full})$
- $u(h_{ind}) = 107 \text{ mm} \ (\text{empty})$

3.1.3.2 Float type

The intrinsic/ measurement uncertainty in the indicated level gauge, $u(h_{ind})$, can be quantified as:
- $u(h_{ind}) = 3.8 \text{ mm} \ [8]$
- $u(h_{ind}) = 4\text{mm} - 8\text{mm} \ [18]$. 
Notes:
- The indicated level may not correspond to the level in the tank due to density effects, i.e., when the density differs in the level gauge pipe (due to temperature gradients) the height of the float is not representative for the level in the tank. There is, however, not data to state something on this possible uncertainty source.
- There is no information available on the Type A uncertainty, however in case ship movement is the main cause for the significant deviation found in the previous section, the Type A uncertainty would be similar for all type of level gauges.

3.1.3.3 Laser type

The intrinsic/measurement uncertainty in the indicated level gauge, \( u(h_{ind}) \), can be quantified as:
- \( u(h_{ind}) = 7.5 \) [18]

Unfortunately there is no statistical information available such that the type A uncertainty cannot be quantified. However, in case ship movement is the main cause for the significant deviation found in the previous section, the Type A uncertainty would be similar for all type of level gauges.

3.1.3.4 Electrical capacitance type

The intrinsic/measurement uncertainty in the indicated level gauge, \( u(h_{ind}) \), can be quantified as:
- \( u(h_{ind}) = 3.8 \text{ mm} \) [8]
- \( u(h_{ind}) = 5 \text{ mm} \) [18]

Unfortunately there is no statistical information available such that the type A uncertainty cannot be quantified. However, in case ship movement is the main cause for the significant deviation found in the previous section, the Type A uncertainty would be similar for all type of level gauges.

3.1.4 Correction tables for list and trim

According to [21] the relative uncertainty in the correction tables for list and trim is equal to the relative uncertainty in the main gauge table, which would imply a standard uncertainty of 0.05 to 0.2%. Next, the corrections for list and trim are assumed to be independent. This may be an accurate assumption for small list and trim angles; however for larger angles a significant correlation may increase the contribution from list and trim on the combined measurement uncertainty.
3.1.5 Calibration and drift in level gauge

The uncertainty due to calibration is because the level gauge equipment is calibrated with finite precision and thus subject to uncertainty. The uncertainty due to drift is because the level gauge equipment is subject to wear which may cause a different reading and thus uncertainty.

Similarly as for the main gauge table however, there is no historical date on calibrations and recalibrations of level gauge equipment. It is, therefore, not possible to quantify this uncertainty source.

3.1.6 Trim

The intrinsic/measurement uncertainty in the indicated trim, \( u(Trim_{ind}) \), can be quantified as:

- \( u(Trim_{ind}) = 0.05m \) [8]
- \( u(Trim_{ind}) = \frac{t}{\sqrt{3}}, t = 0.7\% \) [13]
- \( u(Trim_{ind}) = \frac{t}{\sqrt{3}}, t = 0.06 m \) [13]

In addition to the intrinsic/measurement uncertainty in the indicated trim, there is also a type A uncertainty for the indicated trim. This uncertainty source is, however, yet unknown. The type A uncertainty may be significant, especially for (un)loading at unprotected ports or open sea.

3.1.7 List

The intrinsic/measurement uncertainty in the indicated list, \( u(List_{ind}) \), can be quantified as:

- \( u(List_{ind}) = 0.25^\circ \) [8]
- \( u(List_{ind}) = \frac{t}{\sqrt{3}}, t = 1\% \) [13]
- \( u(List_{ind}) = \frac{t}{\sqrt{3}}, t = 0.03^\circ \) [13]

In addition to the intrinsic/measurement uncertainty in the indicated trim, there is also a type A uncertainty for the indicated trim. This uncertainty source is, however, yet unknown. The type A uncertainty may be significant, especially for (un)loading at unprotected ports or open sea.

3.1.8 Location and calibration of list and drift

Similar as before, there is no historical date on calibrations and recalibrations of the list and trim equipment. It is, therefore, not possible to quantify this uncertainty source.
3.1.9 Mean temperature of the dimensional structure of the tank

3.1.9.1 Moss tanks

The uncertainty in the mean temperature of the dimensional structure of the tank, \( u(T_{\text{tank mean}}) \), can be quantified as:

- \( u(T_{\text{tank mean}}) = 0.13^\circ K \) [8]
- \( u(T_{\text{tank mean}}) = \frac{t}{\sqrt{3}}, t = 0.3 K \) [1]
- \( u(T_{\text{tank mean}}) = 0.5 K \) [11]
- \( u(T_{\text{tank mean}}) = \frac{t}{\sqrt{3}}, t = 0.75 K \) [11]

Note, averaging may imply a larger uncertainty.

3.1.9.2 Membrane tanks

For membrane tanks with supporting structure outside the thermal barrier, the normal reference temperature is 20 °C. However, the temperature of the ballast and seawater has a significant effect on the ambient temperature. As the sea water temperature can significantly differ from place to place, it is plausible that the tank volume changes when a ships moves from a relatively cold to a relatively warm port. Since there is typically no correction for this temperature effect on the supporting structure, the uncertainty in the mean temperature of the dimensional structure of the tank, \( u(T_{\text{tank mean}}) \), is taken as 10 degrees Celsius.

3.1.10 Vapor or gauge temperature

The uncertainty in the level gauge or vapor temperature, \( u(T_{\text{gauge}}) \), can be quantified as:

- \( u(T_{\text{gauge}}) = 0.13^\circ K \) [8]
- \( u(T_{\text{gauge}}) = \frac{t}{\sqrt{3}}, t = 2 K \) [1] [13]
- \( u(T_{\text{gauge}}) = \frac{t}{\sqrt{3}}, t = 5 K \) [8]
- \( u(T_{\text{gauge}}) = \frac{t}{\sqrt{3}}, t = 0.75 K \) [11]

3.1.11 Liquid density and composition

The intrinsic/ measurement uncertainty in the density, \( u(\rho) \), can be quantified as:

- \( u(\rho) = \frac{t}{\sqrt{3}}, t = 0.54\% \) [13]

3.1.12 Material properties tank

The uncertainty in material properties is because these properties are not well known at cryogenic conditions. According to [8] the uncertainty in
the material expansion coefficient for a Moss tank is \( u(\alpha) = 5\% \). The uncertainty in the volumetric temperature expansion coefficient of Membrane type tanks can typically be neglected as Membrane tanks are typically made of invar which hardly expands with increasing temperature.

### 3.1.13 Material properties float level gauge

The uncertainty in the level gauge temperature expansion coefficient, \( u(\beta) \), can be quantified as:

\[
-u(\beta) = \frac{t}{\sqrt{3}}, t = 20\% \quad [8]
\]

### 3.1.14 Volume transferred

In addition to the above discussed uncertainty sources, there are also sources that affect the amount of LNG transferred. Level gauging is performed on predefined time levels, however timing has a significant influence on the measured values. Quoting [18] (section 2.2.1) “As good practice it is recommended that the initial level gauging should be made prior to any cool down operation, i.e. after the (un)loading arms have been connected but before any ship’s liquid and vapor manifolds valves have been opened”.

### 3.2 Uncertainty LNG transferred Membrane type tank

In this section the uncertainty is determined a Membrane type tank and a type radar level gauging. For this example the worst case scenario is assumed. Note, the models and uncertainty budgets developed in this report are based on worst case uncertainty sources as found in the literature and indicated by various experts. In order to have a valid uncertainty budget for a specific LNG carrier, one can use the models and uncertainty budget developed in this report, however one should determine the magnitude of the uncertainty sources for each LNG carrier.

In Table 4 a summary of the uncertainty budget is shown, whereas in Appendix 7 the full uncertainty budget is given, including correlations. The sensitivity multiplied with the uncertainty determines the contribution to the uncertainty in the LNG volume transferred; see also Section 2.1 for a brief explanation on the various terms. The total uncertainty is determined from a quadratic summation of all contributions.

In Table 4 the values given for the various parameters are fictitious, however realistic values for a typical Membrane type tank. The values for the uncertainties are deduced from either the standard uncertainty or the relative standard uncertainty (and is taken from Section 3.1). For example, the relative standard uncertainty for the volume from the main gauge table \( V_{\text{table}} \) is 0.20\%, whereas the standard uncertainty due to sagging and hogging \( \Delta V_{\text{sagging,hogging,star}} \) is 34 (which is 0.1\% of the tank volume). The
uncertainty for the level gauging follows from its own uncertainty budget, see Table 5.

From Table 4 it follows that the most important contribution is from the main gauge table, however also the uncertainty due to sagging and hogging and hydrostatic pressure has a significant influence.

From Table 5 it follows that the uncertainty due to the level gauging is the most important. This uncertainty includes the intrinsic uncertainty as well as the type A uncertainty which has been deduced from the shipment data.

Table 4 Uncertainty budget for a Membrane type tank, see Table 5 for uncertainty budget of the corrected level at start.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>standard uncertainty</th>
<th>relative standard uncertainty</th>
<th>sensitivity</th>
<th>contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{table \ (Trunc(h_{start}))}$</td>
<td>34000</td>
<td>68</td>
<td>0.20%</td>
<td>1</td>
<td>68.0</td>
</tr>
<tr>
<td>$h_{start}$</td>
<td>22.90</td>
<td>0.015</td>
<td>0.07%</td>
<td>-1273</td>
<td>-19.2</td>
</tr>
<tr>
<td>$\Delta V_{SaggingHogging, start}$</td>
<td>0</td>
<td>34</td>
<td>NA</td>
<td>1</td>
<td>34.0</td>
</tr>
<tr>
<td>$\Delta V_{Hydrostatic, start}$</td>
<td>0</td>
<td>34</td>
<td>NA</td>
<td>1</td>
<td>34.0</td>
</tr>
<tr>
<td>$\Delta V_{Table, drift, start}$</td>
<td>0</td>
<td>8.5</td>
<td>NA</td>
<td>1</td>
<td>8.5</td>
</tr>
<tr>
<td>$V_{table \ (Trunc(h_{stop}))}$</td>
<td>1600</td>
<td>3.2</td>
<td>0.20%</td>
<td>1</td>
<td>3.2</td>
</tr>
<tr>
<td>$h_{stop}$</td>
<td>0.15</td>
<td>0.0063</td>
<td>4.19%</td>
<td>-19</td>
<td>-0.1</td>
</tr>
<tr>
<td>$\Delta V_{SaggingHogging, stop}$</td>
<td>0</td>
<td>1.6</td>
<td>NA</td>
<td>1</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Delta V_{Hydrostatic, stop}$</td>
<td>0</td>
<td>1.6</td>
<td>NA</td>
<td>1</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Delta V_{Table, drift, stop}$</td>
<td>0</td>
<td>0.40</td>
<td>NA</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>$T_{tank, start} \ (^{°C})$</td>
<td>20</td>
<td>10</td>
<td>50%</td>
<td>1</td>
<td>11.2</td>
</tr>
<tr>
<td>$T_{tank, stop} \ (^{°C})$</td>
<td>20</td>
<td>10</td>
<td>50%</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$T_{tank, ref} \ (^{°C})$</td>
<td>20</td>
<td>1.0</td>
<td>5.0%</td>
<td>-1</td>
<td>-1.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.10E-05</td>
<td>0.00</td>
<td>5.0%</td>
<td>0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The uncertainty budget previously discussed is based on assuming one tank compartment. However, in general LNG ships have several compartments, which could be considered as different tanks.

In order to determine the uncertainty in the total volume transferred, it is therefore required to know the degree of correlation between the uncertainties per tank. However, most of the various uncertainty sources are highly correlated as they depend on the same measurement equipment, gauge tables and environmental conditions. Therefore, it can be assumed that the uncertainty of the total volume transferred is approximately equal to the uncertainty per tank.
3.3 Uncertainty LNG transferred Moss type tank

In this section the uncertainty is determined in case a LNG shipment is made with a Moss type tank and a type float level gauging. The total uncertainty is determined by means of an uncertainty budget. Also for this example the worst case scenario is assumed.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>standard uncertainty</th>
<th>relative standard uncertainty</th>
<th>sensitivity</th>
<th>contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>h_{ind, start} [m]</td>
<td>22.90</td>
<td>0.014</td>
<td>0.06%</td>
<td>1</td>
<td>0.0135</td>
</tr>
<tr>
<td>Δh_cal [m]</td>
<td>0</td>
<td>0.003</td>
<td>NA</td>
<td>1</td>
<td>0.0025</td>
</tr>
<tr>
<td>Δh_{drift} [m]</td>
<td>0</td>
<td>0.001</td>
<td>NA</td>
<td>1</td>
<td>0.0005</td>
</tr>
<tr>
<td>Δh_{trim, start} [m]</td>
<td>0.007</td>
<td>0</td>
<td>0.10%</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>Trim [m]</td>
<td>0.1</td>
<td>0.030</td>
<td>30%</td>
<td>0.07</td>
<td>0.0021</td>
</tr>
<tr>
<td>c_{Trim, loc, cal} [m]</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>0.07</td>
<td>0.0000</td>
</tr>
<tr>
<td>Δh_{list, start} [m]</td>
<td>-0.012</td>
<td>0</td>
<td>0.10%</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>List [°]</td>
<td>0.5</td>
<td>0.25</td>
<td>50%</td>
<td>-0.023</td>
<td>-0.0058</td>
</tr>
<tr>
<td>c_{List, loc, cal} [°]</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>-0.023</td>
<td>0.0000</td>
</tr>
<tr>
<td>T_{tank, start} [°C]</td>
<td>20</td>
<td>0.50</td>
<td>2.50%</td>
<td>2.51E-04</td>
<td>0.0001</td>
</tr>
<tr>
<td>T_{ref, tank} [°C]</td>
<td>20</td>
<td>0</td>
<td>0.00%</td>
<td>-2.51E-04</td>
<td>0.0000</td>
</tr>
<tr>
<td>T_{gauge, start} [°C]</td>
<td>-160</td>
<td>5.0</td>
<td>-3.13%</td>
<td>-2.29E-05</td>
<td>-0.0001</td>
</tr>
<tr>
<td>T_{ref, level gauge} [°C]</td>
<td>20</td>
<td>0</td>
<td>0%</td>
<td>2.29E-05</td>
<td>0.0000</td>
</tr>
<tr>
<td>α</td>
<td>1.100E-05</td>
<td>0</td>
<td>0.10%</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>β</td>
<td>1.000E-06</td>
<td>0</td>
<td>0.10%</td>
<td>-4113</td>
<td>0.0000</td>
</tr>
<tr>
<td>h_{full}</td>
<td>22.8</td>
<td>u_{h, full}</td>
<td>u_{h, full} *</td>
<td>0.0151</td>
<td>0.0301</td>
</tr>
</tbody>
</table>

In Table 6 a summary of the uncertainty budget is shown, whereas in Appendix 7 the full uncertainty budget is given, including correlations. The sensitivity multiplied with the uncertainty determines the contribution to the uncertainty in the LNG volume transferred; see also Section 2.1 for a brief explanation on the various terms. The total uncertainty is determined from a quadratic summation of all contributions.

In the values given for the various parameters are fictitious, however realistic values for a typical Moss type tank. The values for the uncertainties are deduced from either the standard uncertainty or the relative standard uncertainty (and is taken from Section 3.1).
Table 6 Uncertainty budget for a Moss type tank, see Table 7 for uncertainty budget of the corrected level at start.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>standard uncertainty</th>
<th>relative standard uncertainty</th>
<th>sensitivity</th>
<th>contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{table}}$ (Trunc(h_{\text{start}}))</td>
<td>33198</td>
<td>66</td>
<td>0.20%</td>
<td>1</td>
<td>66.0</td>
</tr>
<tr>
<td>$h_{\text{start}}$</td>
<td>35.000</td>
<td>0.06</td>
<td>0.17%</td>
<td>614</td>
<td>36.0</td>
</tr>
<tr>
<td>$\Delta V_{\text{SaggingHogging, start}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta V_{\text{Hydrostatic, start}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta V_{\text{Table, drift, start}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$V_{\text{table}}$ (Trunc(h_{\text{stop}}))</td>
<td>949</td>
<td>1.90</td>
<td>0.20%</td>
<td>1</td>
<td>1.9</td>
</tr>
<tr>
<td>$h_{\text{stop}}$</td>
<td>4</td>
<td>0.028</td>
<td>0.69%</td>
<td>-459</td>
<td>-12.7</td>
</tr>
<tr>
<td>$\Delta V_{\text{SaggingHogging, stop}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta V_{\text{Hydrostatic, stop}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta V_{\text{Table, drift, stop}}$</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$T_{\text{tank, start}}$ (°C)</td>
<td>-160</td>
<td>3</td>
<td>-1.56%</td>
<td>1.1E+00</td>
<td>2.7</td>
</tr>
<tr>
<td>$T_{\text{tank, stop}}$ (°C)</td>
<td>-160</td>
<td>10</td>
<td>-6.25%</td>
<td>3.1E-02</td>
<td>11.0</td>
</tr>
<tr>
<td>$T_{\text{tank, ref}}$ (°C)</td>
<td>-160</td>
<td>1</td>
<td>-0.63%</td>
<td>-1.1E+00</td>
<td>0.03</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0</td>
<td>5.00%</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_{\text{tank unloaded}}$</td>
<td>39179</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7 Uncertainty budget for the corrected level at start.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>standard uncertainty</th>
<th>relative standard uncertainty</th>
<th>sensitivity</th>
<th>contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{\text{ind, start}}$ [m]</td>
<td>35.000</td>
<td>0.014</td>
<td>0.04%</td>
<td>1</td>
<td>0.014</td>
</tr>
<tr>
<td>$\Delta h_{\text{cal}}$ [m]</td>
<td>0</td>
<td>0.003</td>
<td>NA</td>
<td>1</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\Delta h_{\text{drift}}$ [m]</td>
<td>0</td>
<td>0.01</td>
<td>NA</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta h_{\text{trim, start}}$ [m]</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Trim [m]</td>
<td>0</td>
<td>0.03</td>
<td>NA</td>
<td>-0.1</td>
<td>-0.003</td>
</tr>
<tr>
<td>$c_{\text{Trim, loc, cal}}$ [m]</td>
<td>0</td>
<td>0.05</td>
<td>NA</td>
<td>-0.1</td>
<td>-0.005</td>
</tr>
<tr>
<td>$\Delta h_{\text{list, start}}$ [m]</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>List [°]</td>
<td>0</td>
<td>0.5</td>
<td>NA</td>
<td>-0.1</td>
<td>-0.05</td>
</tr>
<tr>
<td>$c_{\text{List, loc, cal}}$ [°]</td>
<td>0</td>
<td>0.25</td>
<td>NA</td>
<td>-0.1</td>
<td>-0.025</td>
</tr>
<tr>
<td>$T_{\text{tank, start}}$ [°C]</td>
<td>-160</td>
<td>0.25</td>
<td>-0.16%</td>
<td>6.3E-04</td>
<td>0.0002</td>
</tr>
<tr>
<td>$T_{\text{ref, tank}}$ [°C]</td>
<td>-160</td>
<td>0</td>
<td>0%</td>
<td>-6.3E-04</td>
<td>0</td>
</tr>
<tr>
<td>$T_{\text{gauge, start}}$ [°C]</td>
<td>-140</td>
<td>2.5</td>
<td>-1.79%</td>
<td>-3.2E-05</td>
<td>-0.0001</td>
</tr>
<tr>
<td>$T_{\text{ref, level gauge}}$ [°C]</td>
<td>20</td>
<td>0.5</td>
<td>2.50%</td>
<td>3.2E-05</td>
<td>0.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.8E-05</td>
<td>0.0</td>
<td>10%</td>
<td>0</td>
<td>0.0</td>
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<tr>
<td>$\beta$</td>
<td>9.0E-07</td>
<td>0.0</td>
<td>10%</td>
<td>-5600</td>
<td>-0.0005</td>
</tr>
<tr>
<td>$h_{\text{full}}$</td>
<td>35.005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$U_{V, \text{loaded}}$ 74.93

$U_{V, \text{loaded}}^*$ 149.86

$U_{V, \text{loaded}}^*$ 0.38%
The uncertainty for the level gauging follows from its own uncertainty budget, see Table 7. From Table 6 it follows that the most important contribution is from the main gauge table, however also the uncertainty due to sagging and hogging and hydrostatic pressure has a significant influence. From Table 7 it follows that the uncertainty due to the level gauging is the most important. This uncertainty includes the intrinsic uncertainty as well as the type A uncertainty which has been deduced from the shipment data.

4 Conclusions

The goal of this report is to evaluate the uncertainty model adopted in [1] and to validate the uncertainty figure of the (un)loaded LNG volume. In order to achieve this goal various experts in the field of LNG shipping have been interviewed. In addition, a significant data set of LNG (un)loadings has been collected and analyzed.

Unlike other uncertainty estimations that can be found in the literature, the one presented in this work is fully in accordance with the GUM and it includes real shipment data. In addition, in this report the formulas used to determine the uncertainty associated with the LNG volume transferred to or from a ship. The results are applicable to both Moss type and Membrane type tanks.

This study indicates that the uncertainty in level gauging is higher, potentially much higher, than stated elsewhere. For a Membrane type tank, for example, the total uncertainty is significantly higher than stated in the GIIGNL LNG custody transfer handbook. In case the differences in level gauging equipment are taken into account, the uncertainty is close to 1%. Otherwise, the largest uncertainty contribution comes from the main gauge table.

Uncertainty contributions from trim and list are essential for terminals that are poorly protected from or are at open sea.

5 Recommendations

This work has gone some way towards enhancing our understanding of the various contributions that affect the proper determination of an LNG volume loaded/unloaded to/from a ship. But to achieve even more confidence around the prepared uncertainty estimations it is recommended that further research be undertaken in the following areas:

The need to dedicate the described model to a certain ship as the overall uncertainty may be significant different for each ship.
Displaced gas and dead volumes are not taken into account in the LNG volume determination. Although this does not add to the uncertainty for the LNG tank volume determination, it does add to the uncertainty of the LNG volume transferred from buyer to seller. As there may be a significant temperature difference, this may be an important uncertainty factor. Furthermore, static LNG volume determinations are inappropriate when the LNG tanks have to be purged and/or cooled down.

Human errors have not been taken into account in the uncertainty budget as they can be avoided when the procedures and protocols are followed. However, there are various examples where human errors can significantly increase the overall uncertainty. Hence, a proper assessment of human errors should be part of the uncertainty budget.

The shipment data presented may indicate that level gauging uncertainty is much larger than what is typically believed. However, further study is required as the shipment data is significantly small.

Finally, some of the sensitivities depend on the ship conditions (trim and list). Typically, the uncertainty increases when the list or trim increases. Hence, different ship conditions should be studied to get a deeper understanding of the total uncertainty. This is especially important for LNG (un)loading at open sea.

6 References

6.1 Norms and certificates


### 6.2 Technical reports and articles


### 6.3 Other


[22] Sele, A. (Chief Engineer at Aker Engineering & Technology AS), email conversation, December 2010.


7  Appendix: full uncertainty budget  Membrane type tank

Table 8  Full uncertainty budget for a Membrane type tank including correlations.

<table>
<thead>
<tr>
<th>Measurand</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Distribution</th>
<th>Standard uncertainty</th>
<th>Nat. st. uncertainty</th>
<th>Sensitivity</th>
<th>Contribution</th>
<th>TABLE 8 Uncertainty budget for Membrane type tank including correlations.</th>
<th>Vtable (Trunc(hstart))</th>
<th>hstart</th>
<th>DV_Sagging_Hogging, start</th>
<th>DV_Hydrostatic, start</th>
<th>DV_Table, drift, start</th>
<th>Vtable (Trunc(hstop))</th>
<th>hstop</th>
<th>DV_Sagging_Hogging, stop</th>
<th>DV_Hydrostatic, stop</th>
<th>DV_Table, drift, stop</th>
<th>Tank, start</th>
<th>Tank, stop</th>
<th>Tank, off</th>
<th>α</th>
<th>Vtank unloaded</th>
<th>U_{V_{\text{loaded}}}</th>
<th>U_{V_{\text{loaded}}} *</th>
<th>U_{\alpha}</th>
</tr>
</thead>
<tbody>
<tr>
<td>hstart</td>
<td>23.00</td>
<td>0.01507</td>
<td>standard</td>
<td>0.01507</td>
<td>0.07%</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>ΔV_{Sagging_Hogging}</td>
<td>0.000</td>
<td>68.000</td>
<td>rectangular</td>
<td>34.000</td>
<td>NA</td>
<td>1</td>
<td>34.00</td>
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</tr>
<tr>
<td>ΔV_{Table, drift}</td>
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<td>NA</td>
<td>1</td>
<td>8.500</td>
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<td>hstop</td>
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<td>standard</td>
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<td>-1273</td>
<td>-19.19</td>
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<td>ΔV_{Sagging_Hogging}</td>
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<td>rectangular</td>
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<td>NA</td>
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<tr>
<td>ΔV_{Table, drift}</td>
<td>0.000</td>
<td>0.800</td>
<td>rectangular</td>
<td>0.400</td>
<td>NA</td>
<td>1</td>
<td>0.4</td>
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<td></td>
</tr>
<tr>
<td>T_{tank, start} (°C)</td>
<td>20.00</td>
<td>10.000</td>
<td>rectangular</td>
<td>5.000</td>
<td>1</td>
<td>11.22</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>T_{tank, stop} (°C)</td>
<td>20.00</td>
<td>10.000</td>
<td>rectangular</td>
<td>5.000</td>
<td>1</td>
<td>11.22</td>
<td>0</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_{tank, off} (°C)</td>
<td>20.00</td>
<td>5.000</td>
<td>rectangular</td>
<td>2.500</td>
<td>1</td>
<td>-1</td>
<td>-1.12</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α</td>
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<td>1.1E-06</td>
<td>rectangular</td>
<td>0.000</td>
<td>5.00%</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correlation matrix

<table>
<thead>
<tr>
<th>Value</th>
<th>Vtable (Trunc(hstart))</th>
<th>hstart</th>
<th>DV_Sagging_Hogging, start</th>
<th>DV_Hydrostatic, start</th>
<th>DV_Table, drift, start</th>
<th>Vtable (Trunc(hstop))</th>
<th>hstop</th>
<th>DV_Sagging_Hogging, stop</th>
<th>DV_Hydrostatic, stop</th>
<th>DV_Table, drift, stop</th>
<th>Tank, start</th>
<th>Tank, stop</th>
<th>Tank, off</th>
<th>α</th>
<th>Vtank unloaded</th>
<th>U_{V_{\text{loaded}}}</th>
<th>U_{V_{\text{loaded}}} *</th>
<th>U_{\alpha}</th>
</tr>
</thead>
<tbody>
<tr>
<td>34000</td>
<td>196.50</td>
<td>35506</td>
<td>0.00</td>
<td>0.0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>33242.13</td>
<td>87.97</td>
<td>175.95</td>
<td>0.53%</td>
</tr>
</tbody>
</table>

...
### Table 9: Full uncertainty budget for a radar type level gauging before unloading

| meas                  | val     | unc    | distr | std unc | rel unc | sens | contr | meas                  | val     | unc    | distr | std unc | rel unc | sens | contr | meas                  | val     | unc    | distr | std unc | rel unc | sens | contr | meas                  | val     | unc    | distr | std unc | rel unc | sens | contr | meas                  | val     | unc    |
|-----------------------|---------|--------|-------|---------|---------|------|-------|-----------------------|---------|--------|-------|---------|---------|------|-------|-----------------------|---------|--------|-------|---------|---------|------|-------|-----------------------|---------|--------|-------|---------|---------|------|-------|-----------------------|---------|--------|-------|---------|---------|------|-------|-----------------------|---------|--------|
| $h_{start}$ [m]       | 22.850  | 0.027  | normal| 0.014   | 0.06%   | 1    | 0.0135| $\Delta h_{cal}$ [m]   | 0.000   | 0.005  | rectangular | 0.003 | NA    | 1    | 0.0025 | 0        | 0    | 0      | $\Delta h_{trim}$ [m]    | 0.007   | 0.0001 | normal | 0.000   | 0.10%   | 1    | 0.0005 | $\Delta h_{list}$ [m]   | -0.012  | -2.33E-05 | rectangular | 0.000   | 0.10% | 1    | 0.0000 | 0        | 0    | 0      |
|                      |         |        |       |         |         |      |       | $c_{Trim,loc,cal}$ [m]  | 0.000   | 0      | normal    | 0.000   | 0.250  |      | -0.023  | $\Delta h_{list,stat}$ [m] | -0.012  | 0.10%  | 1      | 0.0000 | 0        | 0    | 0      | List [°]               | 0.500   | 0.5    | normal | 0.000   | 50.00%  | -0.023 | -0.0058 |                      |                      |         |         |         |         |         |      |       |
| $c_{List,loc,cal}$ [°] | 0.000   | 0      | normal| 0.000   | NA      | -0.023| 0.0000| $T_{tank,start}$ [°C]   | 20.000  | 0.5    | standard | 0.500   | 2.50%  | 2.51E-04 | 0.0001 | 0  | 0      | $T_{ref,tank}$ [°C]      | 20.000  | 0      | standard | 0.000   | 0.00%  | -2.51E-04 | 0.0000 | 0  | 0      | $T_{gauge,start}$ [°C]   | -160.000 | 5    | standard | 5.000   | -3.13% | -2.29E-05 | -0.0001 | 0  | 0      |                     |                      |         |         |         |         |         |      |       |
|                       |         |        |       |         |         |      |       | $T_{ref,level gauge}$ [°C] | 20.000  | 0    | standard | 0.000   | 0.00%  | 2.29E-05 | 0.0000 | 0  | 0      | $\alpha$                           | 1.100E-05 | 1.000E-08 | standard | 0.000   | 0.10%  | 0      | 0.0000 | 0        | 0    | 0      |
|                       |         |        |       |         |         |      |       | $\beta$                           | 1.000E-06 | 1.000E-09 | standard | 0.000   | 0.10%  | -4113  | 0.0000 | 0        | 0    | 0      |
| $h_{full}$            | 22.849  |        |       |         |         |      |       | $U_{h_{full}}$               | 0.151  |        |         |         |         |      |       | $U_{h_{full}}$               | 0.0301  |        |         |         |         |      |       |
|                       |         |        |       |         |         |      |       | $U_{h_{full}}^*$              | 0.13%  |        |         |         |         |      |       |                      |                      |         |         |         |         |         |      |       |

The table outlines the uncertainty budget for a radar type level gauging before unloading, detailing measurements, uncertainties, and their respective contributions.
### Table 10: Full uncertainty budget for a radar type level gauging after unloading

<table>
<thead>
<tr>
<th>Measurand</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Distribution</th>
<th>Standard uncertainty</th>
<th>Rel. Uncertainty</th>
<th>Sensitivity</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{stop}} ) [m]</td>
<td>0.150</td>
<td>0.0090</td>
<td>normal</td>
<td>0.00</td>
<td>3.00%</td>
<td>1</td>
<td>0.0045</td>
</tr>
<tr>
<td>( \Delta h_{\text{cal}} ) [m]</td>
<td>0.000</td>
<td>0.005</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0025</td>
</tr>
<tr>
<td>( \Delta h_{\text{drift}} ) [m]</td>
<td>0.000</td>
<td>0.001</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0005</td>
</tr>
<tr>
<td>( \Delta h_{\text{trim,stop}} ) [m]</td>
<td>0.000</td>
<td>0</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>Trim [m]</td>
<td>0.000</td>
<td>0.06</td>
<td>normal</td>
<td>0.03</td>
<td>NA</td>
<td>0.070</td>
<td>0.0021</td>
</tr>
<tr>
<td>( \Delta \text{temperature, cal} ) [m]</td>
<td>0.000</td>
<td>0</td>
<td>normal</td>
<td>0.00</td>
<td>NA</td>
<td>0.070</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \Delta T_{\text{tank,stop}} ) [°C]</td>
<td>20.000</td>
<td>0</td>
<td>standard</td>
<td>0.00</td>
<td>-1.650E-06</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \Delta T_{\text{gauge,stop}} ) [°C]</td>
<td>-130.000</td>
<td>5</td>
<td>rectangular</td>
<td>2.50</td>
<td>-1.350E-07</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.100E-05</td>
<td>1.1E-08</td>
<td>standard</td>
<td>0.00</td>
<td>1.350E-07</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \beta )</td>
<td>9.000E-07</td>
<td>9E-10</td>
<td>standard</td>
<td>0.00</td>
<td>9.00E-07</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Correlation matrix**

<table>
<thead>
<tr>
<th>Measurand</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Distribution</th>
<th>Standard uncertainty</th>
<th>Rel. Uncertainty</th>
<th>Sensitivity</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{stop}} )</td>
<td>0.150</td>
<td>0.0090</td>
<td>normal</td>
<td>0.00</td>
<td>3.00%</td>
<td>1</td>
<td>0.0045</td>
</tr>
<tr>
<td>( \Delta h_{\text{cal}} )</td>
<td>0.000</td>
<td>0.005</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0025</td>
</tr>
<tr>
<td>( \Delta h_{\text{drift}} )</td>
<td>0.000</td>
<td>0.001</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0005</td>
</tr>
<tr>
<td>( \Delta h_{\text{trim,stop}} )</td>
<td>0.000</td>
<td>0</td>
<td>rectangular</td>
<td>0.00</td>
<td>NA</td>
<td>1</td>
<td>0.0000</td>
</tr>
<tr>
<td>Trim [m]</td>
<td>0.000</td>
<td>0.06</td>
<td>normal</td>
<td>0.03</td>
<td>NA</td>
<td>0.070</td>
<td>0.0021</td>
</tr>
<tr>
<td>( \Delta \text{temperature, cal} )</td>
<td>0.000</td>
<td>0</td>
<td>normal</td>
<td>0.00</td>
<td>NA</td>
<td>0.070</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \Delta T_{\text{tank,stop}} )</td>
<td>20.000</td>
<td>0</td>
<td>standard</td>
<td>0.00</td>
<td>-1.650E-06</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \Delta T_{\text{gauge,stop}} )</td>
<td>-130.000</td>
<td>5</td>
<td>rectangular</td>
<td>2.50</td>
<td>-1.350E-07</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.100E-05</td>
<td>1.1E-08</td>
<td>standard</td>
<td>0.00</td>
<td>1.350E-07</td>
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</tr>
<tr>
<td>( \beta )</td>
<td>9.000E-07</td>
<td>9E-10</td>
<td>standard</td>
<td>0.00</td>
<td>9.00E-07</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Notes:**
- \( h_{\text{stop}} \) is the stop height.
- \( U_{\text{h,stop}} \) is the uncertainty of \( h_{\text{stop}} \).
- \( U_{\text{h,stop}}^* \) is the expanded uncertainty of \( h_{\text{stop}} \).